



# 第七章 FIR数字滤波器的设计方法

IIR数字滤波器:

可以利用模拟滤波器设计  
但相位非线性

FIR数字滤波器:

可以严格线性相位, 又可任意幅度特性  
因果稳定系统  
可用FFT计算  
但阶次比IIR滤波器要高得多

# 一、线性相位FIR滤波器的特点

FIR滤波器的单位冲激响应:

$$h(n) \quad 0 \leq n \leq N-1$$

系统函数:

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}$$

在  $z$  平面有  $N-1$  个零点

在  $z=0$  处是  $N-1$  阶极点



# 1、线性相位条件

$h(n)$ 为实序列时，其频率响应：

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n} = H(\omega)e^{j\theta(\omega)} = \pm |H(e^{j\omega})| e^{j\theta(\omega)}$$

线性相位是指  $\theta(\omega)$  是  $\omega$  的线性函数

即群延时  $-\frac{d\theta(\omega)}{d\omega} = \tau$  是常数

第一类线性相位： $\theta(\omega) = -\tau\omega$

第二类线性相位： $\theta(\omega) = \beta_0 - \tau\omega$

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n} = \pm |H(e^{j\omega})| e^{j\theta(\omega)} = \pm |H(e^{j\omega})| e^{-j\omega\tau}$$

第一类线性相位：  $\theta(\omega) = -\tau\omega$

$$\pm |H(e^{j\omega})| \cos(\omega\tau) = \sum_{n=0}^{N-1} h(n) \cos(\omega n)$$

$$\pm |H(e^{j\omega})| \sin(\omega\tau) = \sum_{n=0}^{N-1} h(n) \sin(\omega n)$$

$$\text{tg}(\omega\tau) = \frac{\sin(\omega\tau)}{\cos(\omega\tau)} = \frac{\sum_{n=0}^{N-1} h(n) \sin(\omega n)}{\sum_{n=0}^{N-1} h(n) \cos(\omega n)}$$

$$\sum_{n=0}^{N-1} h(n) \sin(\omega\tau) \cos(\omega n) - \sum_{n=0}^{N-1} h(n) \cos(\omega\tau) \sin(\omega n) = 0$$

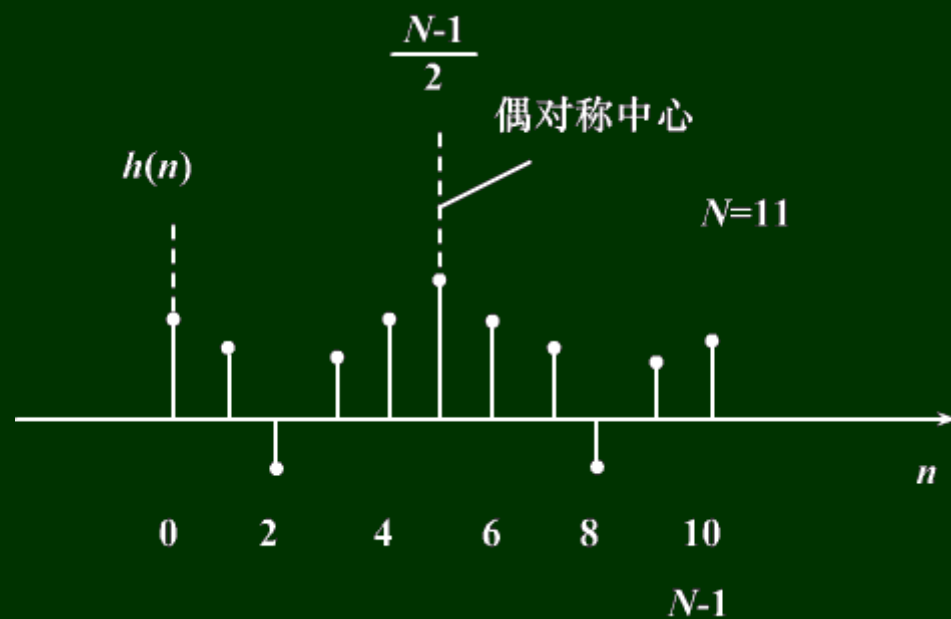
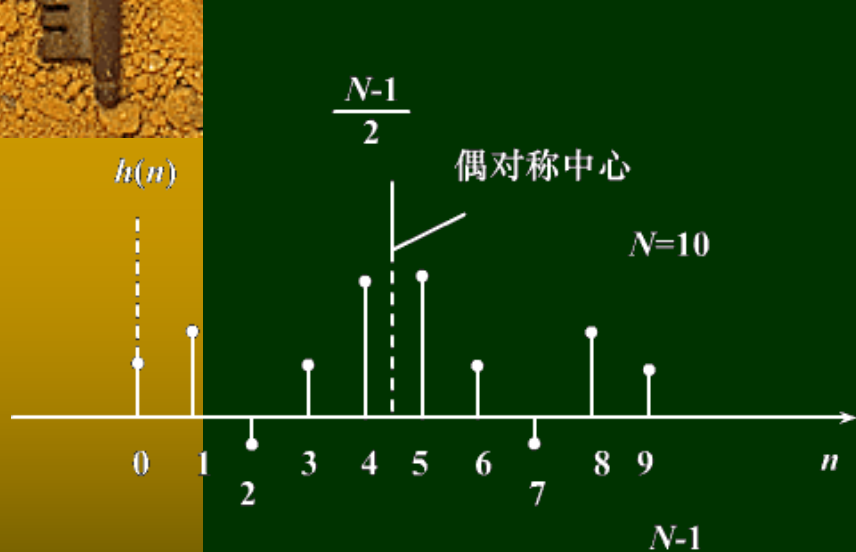
$$\sum_{n=0}^{N-1} h(n) \sin[(\tau - n)\omega] = 0$$



- ◆ 第一类线性相位  $\theta(\omega) = -\tau\omega$  的充要条件:

$$h(n) = h(N-1-n) \quad 0 \leq n \leq N-1$$

$$n = (N-1)/2 \text{ 为 } h(n) \text{ 的偶对称中心} \quad \tau = \frac{N-1}{2}$$



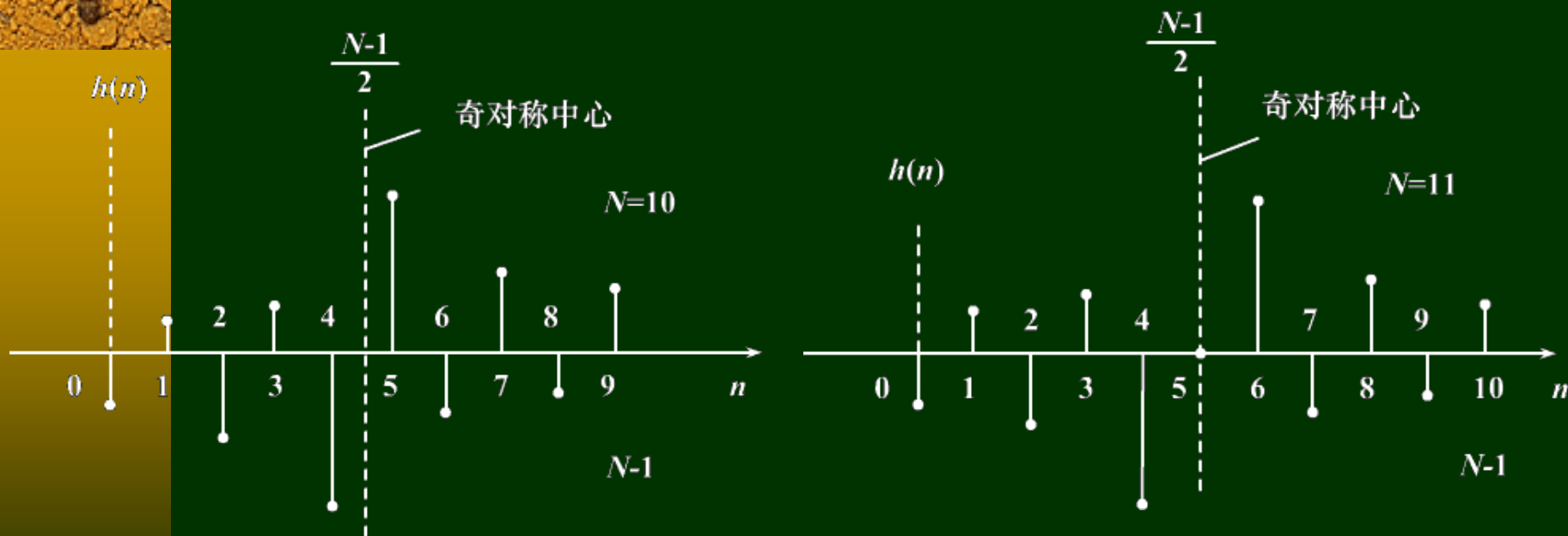
$$\sum_{n=0}^{N-1} h(n) \sin[(\tau - n)\omega] = 0$$

◆ 第二类线性相位  $\theta(\omega) = \beta_0 - \tau\omega$  的充要条件:

$$h(n) = -h(N-1-n) \quad 0 \leq n \leq N-1$$

$$n = (N-1)/2 \text{ 为 } h(n) \text{ 的奇对称中心} \quad \tau = \frac{N-1}{2}$$

$$\beta_0 = \pm\pi/2$$



## 2、线性相位FIR滤波器频率响应的特点

由  $h(n) = \pm h(N-1-n) \quad 0 \leq n \leq N-1$

系统函数:

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n} = \sum_{n=0}^{N-1} \pm h(N-1-n)z^{-n}$$


$$\text{令 } m = N-1-n$$

$$= \sum_{m=0}^{N-1} \pm h(m)z^{-(N-1-m)}$$

$$= \pm z^{-(N-1)} \sum_{m=0}^{N-1} h(m)z^m$$

$$= \pm z^{-(N-1)} H(z^{-1})$$





由  $H(z) = \pm z^{-(N-1)} H(z^{-1})$

得  $H(z) = \frac{1}{2} \left[ H(z) \pm z^{-(N-1)} H(z^{-1}) \right]$

$$= \frac{1}{2} \left[ \sum_{n=0}^{N-1} h(n) z^{-n} \pm z^{-(N-1)} \sum_{n=0}^{N-1} h(n) z^n \right]$$

$$= \frac{1}{2} \sum_{n=0}^{N-1} h(n) \left[ z^{-n} \pm z^{-(N-1)} z^n \right]$$

$$= z^{-\frac{N-1}{2}} \sum_{n=0}^{N-1} h(n) \left[ \frac{z^{\left(\frac{N-1}{2}-n\right)} \pm z^{-\left(\frac{N-1}{2}-n\right)}}{2} \right]$$





$$H(z) = z^{-\frac{N-1}{2}} \sum_{n=0}^{N-1} h(n) \left[ \frac{z^{\left(\frac{N-1}{2}-n\right)} \pm z^{-\left(\frac{N-1}{2}-n\right)}}{2} \right]$$

$$\frac{z^{\left(\frac{N-1}{2}-n\right)} \pm z^{-\left(\frac{N-1}{2}-n\right)}}{2} \Big|_{z=e^{j\omega}} = \begin{cases} \cos \left[ \left( \frac{N-1}{2} - n \right) \omega \right] & "+" \\ j \sin \left[ \left( \frac{N-1}{2} - n \right) \omega \right] & "-" \end{cases}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$H(e^{j\omega}) = H(z) \Big|_{z=e^{j\omega}} = \begin{cases} e^{-j\frac{N-1}{2}\omega} \sum_{n=0}^{N-1} h(n) \cos \left[ \left( \frac{N-1}{2} - n \right) \omega \right] & "+" \\ j e^{-j\frac{N-1}{2}\omega} \sum_{n=0}^{N-1} h(n) \sin \left[ \left( \frac{N-1}{2} - n \right) \omega \right] & "-" \end{cases}$$

1)  $h(n)$  偶对称

$$h(n) = h(N-1-n)$$

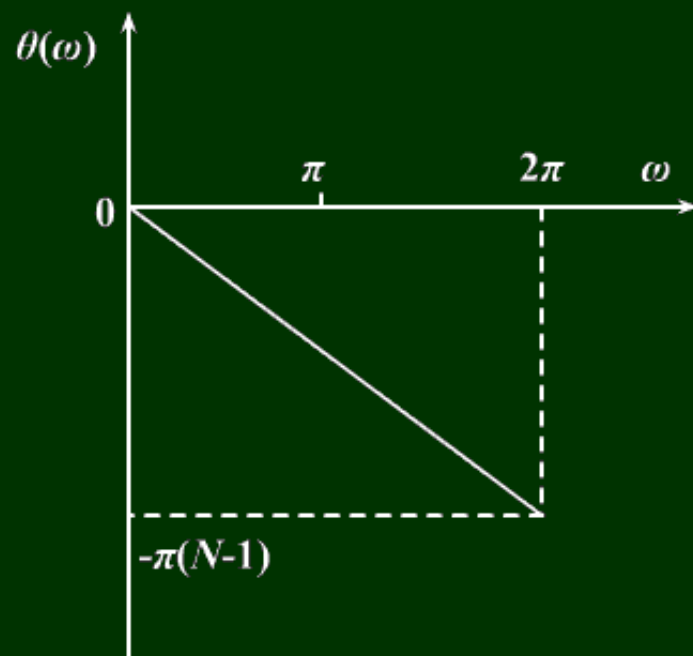
◆ 频率响应:

$$H(e^{j\omega}) = H(z) \Big|_{z=e^{j\omega}} = e^{-j\frac{N-1}{2}\omega} \sum_{n=0}^{N-1} h(n) \cos \left[ \left( \frac{N-1}{2} - n \right) \omega \right]$$

相位函数:  $\theta(\omega) = -\frac{N-1}{2}\omega$

为第一类线性相位

$$\tau = \frac{N-1}{2}$$



## 2) $h(n)$ 奇对称

$$h(n) = -h(N-1-n)$$

◆ 频率响应:

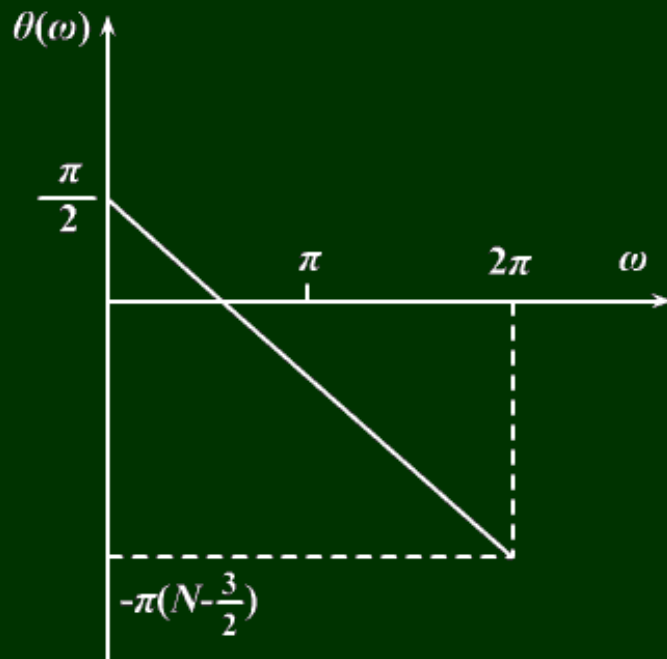
$$\begin{aligned} H(e^{j\omega}) &= H(z) \Big|_{z=e^{j\omega}} = je^{-j\frac{N-1}{2}\omega} \sum_{n=0}^{N-1} h(n) \sin \left[ \left( \frac{N-1}{2} - n \right) \omega \right] \\ &= e^{-j\frac{N-1}{2}\omega + j\frac{\pi}{2}} \sum_{n=0}^{N-1} h(n) \sin \left[ \left( \frac{N-1}{2} - n \right) \omega \right] \end{aligned}$$

相位函数:

$$\theta(\omega) = -\frac{N-1}{2}\omega + \frac{\pi}{2}$$

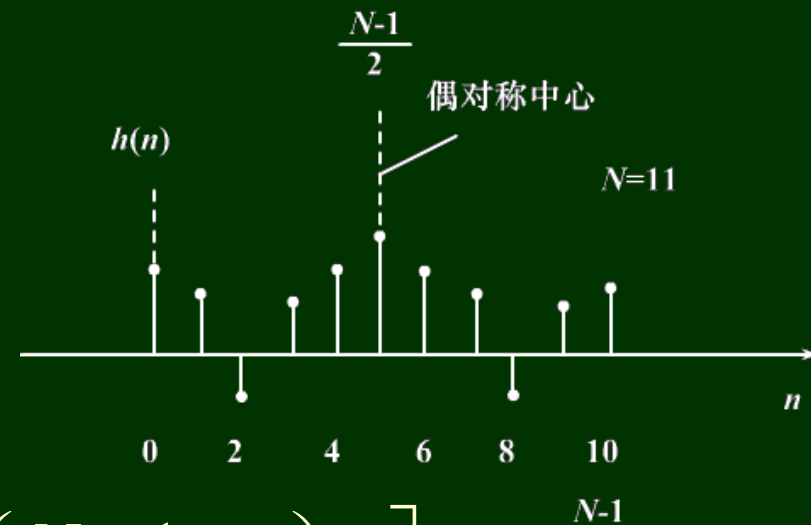
为第二类线性相位

$$\tau = \frac{N-1}{2} \quad \beta_0 = \pi/2$$



### 3、幅度函数的特点

1)  $h(n)$  偶对称,  $N$  为奇数



幅度函数:

$$H(\omega) = \sum_{n=0}^{N-1} h(n) \cos \left[ \left( \frac{N-1}{2} - n \right) \omega \right]$$

$$\because \cos \left\{ \left[ \frac{N-1}{2} - (N-1-n) \right] \omega \right\} = \cos \left[ \left( n - \frac{N-1}{2} \right) \omega \right]$$

$$= \cos \left[ \left( \frac{N-1}{2} - n \right) \omega \right]$$

$$\therefore \cos \left[ \left( \frac{N-1}{2} - n \right) \omega \right] \text{ 对 } \frac{N-1}{2} \text{ 呈偶对称}$$

$$H(\omega) = h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} 2h(n) \cos\left[\left(\frac{N-1}{2} - n\right)\omega\right]$$

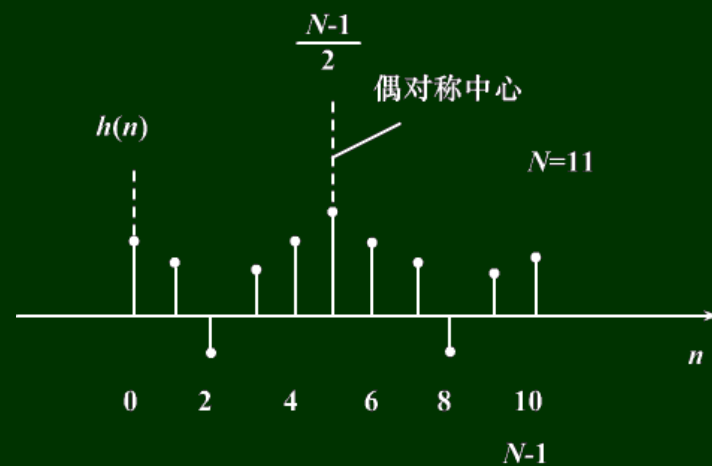
$$\text{令 } \frac{N-1}{2} - n = m$$

$$= h\left(\frac{N-1}{2}\right) + \sum_{m=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - m\right) \cos(m\omega)$$

$$\therefore H(\omega) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos(\omega n)$$

其中：  $a(0) = h\left(\frac{N-1}{2}\right)$

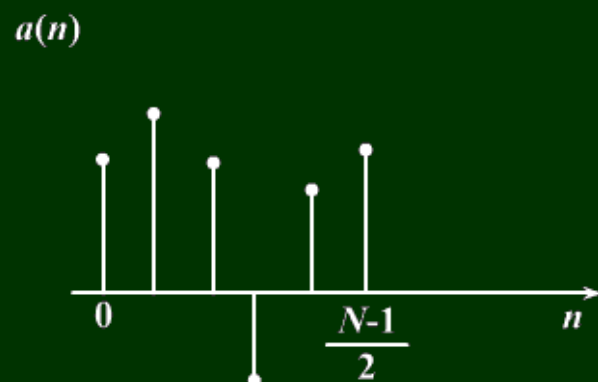
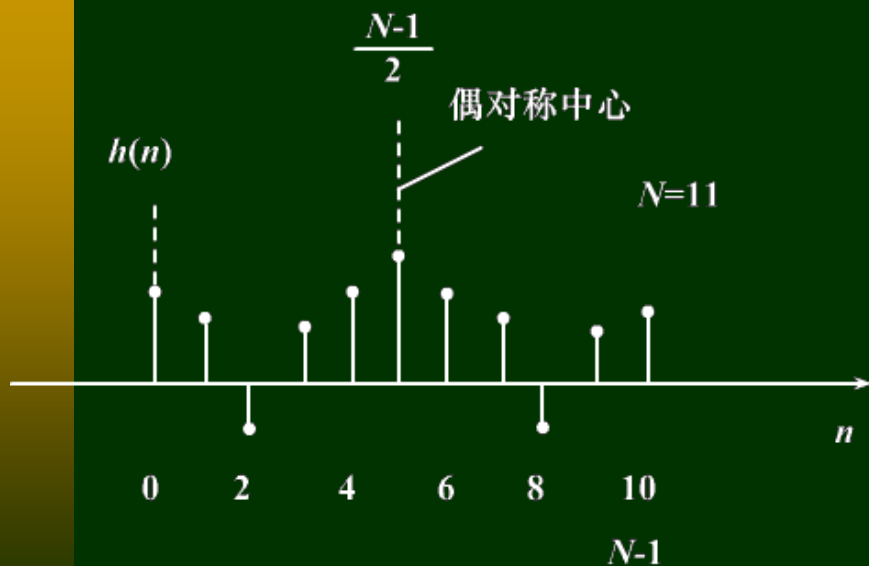
$$a(n) = 2h\left(\frac{N-1}{2} - n\right) \quad n = 1, \dots, \frac{N-1}{2}$$




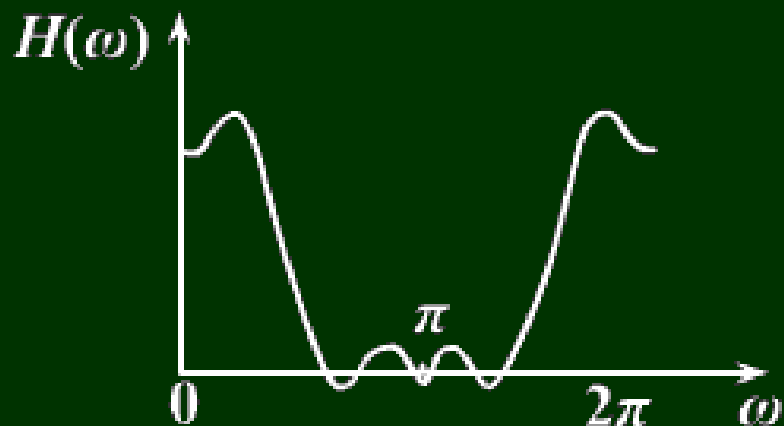
$$H(\omega) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos(\omega n)$$

其中:  $a(0) = h\left(\frac{N-1}{2}\right)$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right) \quad n = 1, \dots, \frac{N-1}{2}$$



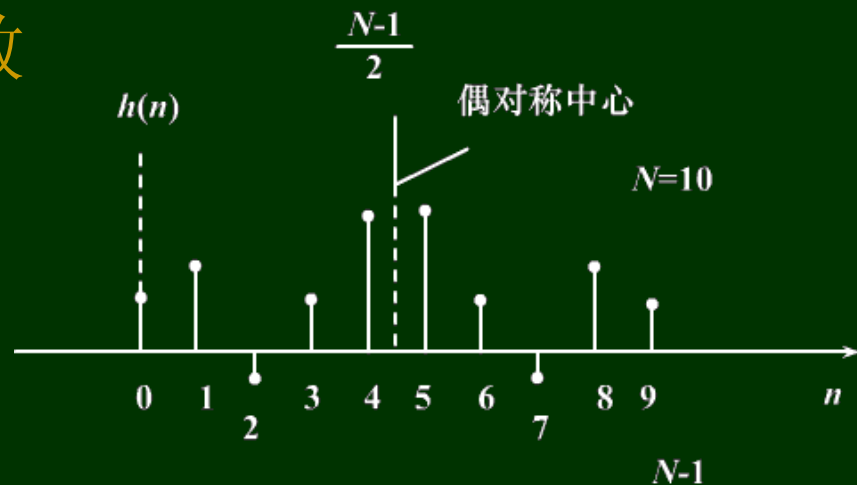

$$H(\omega) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos(\omega n)$$



$\because \cos(\omega n)$ 对  $\omega = 0, \pi, 2\pi$  呈偶对称

$\therefore H(\omega)$ 对  $\omega = 0, \pi, 2\pi$  呈偶对称

## 2) $h(n)$ 偶对称, $N$ 为偶数



幅度函数:

$$H(\omega) = \sum_{n=0}^{N-1} h(n) \cos \left[ \left( \frac{N-1}{2} - n \right) \omega \right]$$

$$= \sum_{n=0}^{\frac{N}{2}-1} 2h(n) \cos \left[ \left( \frac{N-1}{2} - n \right) \omega \right]$$





$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} 2h(n) \cos \left[ \left( \frac{N-1}{2} - n \right) \omega \right]$$

$$\text{令 } \frac{N}{2} - n = m$$

$$= \sum_{m=1}^{\frac{N}{2}} 2h \left( \frac{N}{2} - m \right) \cos \left[ \left( m - \frac{1}{2} \right) \omega \right]$$

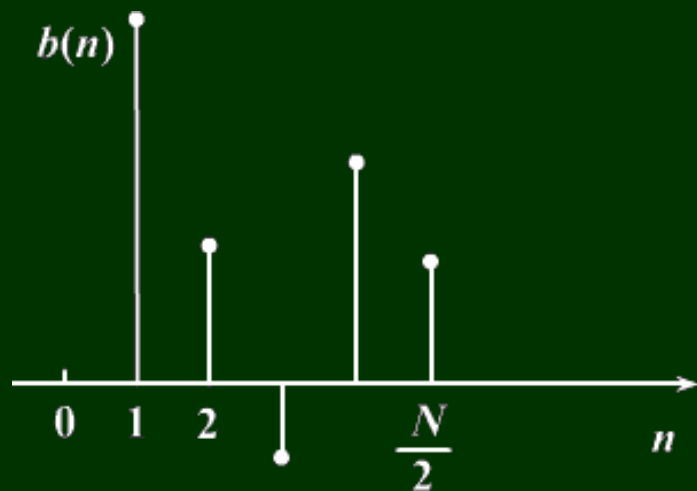
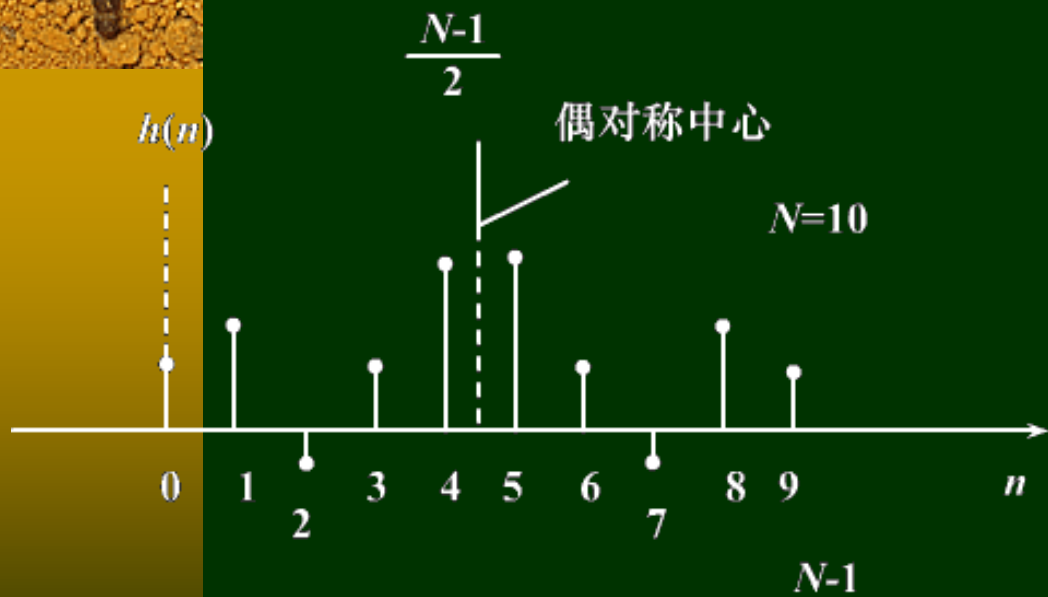
$$\therefore H(\omega) = \sum_{n=1}^{N/2} b(n) \cos \left[ \omega \left( n - \frac{1}{2} \right) \right]$$


$$\text{其中: } b(n) = 2h \left( \frac{N}{2} - n \right) \quad n = 1, \dots, \frac{N}{2}$$

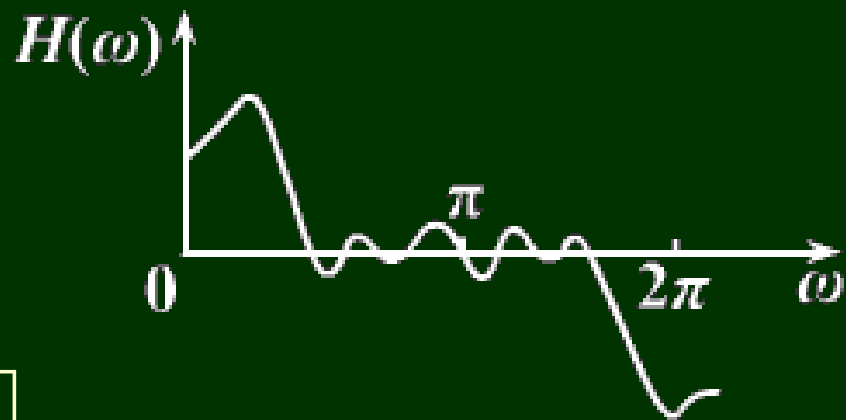


$$H(\omega) = \sum_{n=1}^{N/2} b(n) \cos \left[ \omega \left( n - \frac{1}{2} \right) \right]$$

其中： $b(n) = 2h \left( \frac{N}{2} - n \right)$        $n = 1, \dots, \frac{N}{2}$




$$H(\omega) = \sum_{n=1}^{N/2} b(n) \cos \left[ \omega \left( n - \frac{1}{2} \right) \right]$$



◆  $\omega = \pi$  时  $\cos \left[ \omega \left( n - \frac{1}{2} \right) \right] = 0$

则  $H(\pi) = 0$   $\therefore z = -1$  是零点

◆  $H(\omega)$  对  $\omega = 0, 2\pi$  呈偶对称  $H(\omega)$  对  $\omega = \pi$  呈奇对称

◆  $z = -1$  为零点 故不能设计成高通、带阻滤波器

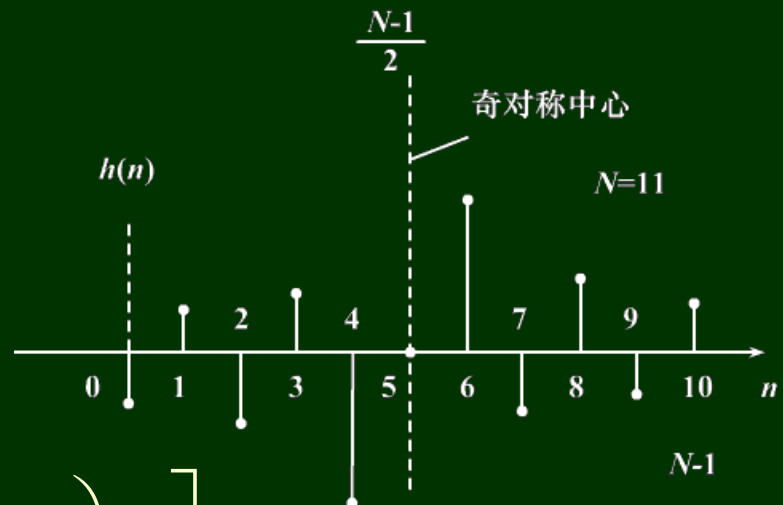
### 3) $h(n)$ 奇对称, $N$ 为奇数

幅度函数:

$$H(\omega) = \sum_{n=0}^{N-1} h(n) \sin \left[ \left( \frac{N-1}{2} - n \right) \omega \right]$$

$$\begin{aligned} \because \sin \left\{ \left[ \frac{N-1}{2} - (N-1-n) \right] \omega \right\} &= \sin \left[ \left( n - \frac{N-1}{2} \right) \omega \right] \\ &= -\sin \left[ \left( \frac{N-1}{2} - n \right) \omega \right] \end{aligned}$$

$$\therefore \sin \left[ \left( \frac{N-1}{2} - n \right) \omega \right] \text{ 对 } \frac{N-1}{2} \text{ 呈奇对称}$$



$h(n)$  奇对称且  $N$  为奇数  $\therefore h\left(\frac{N-1}{2}\right) = 0$

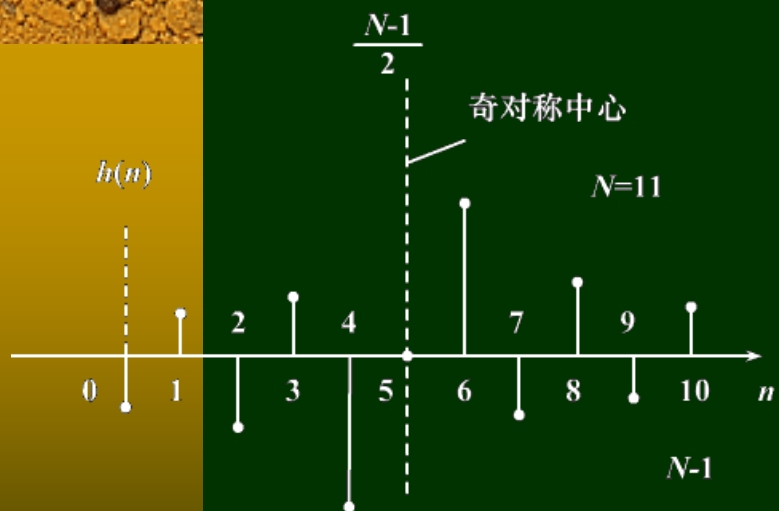
$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} 2h(n) \sin\left[\left(\frac{N-1}{2} - n\right)\omega\right]$$

令  $\frac{N-1}{2} - n = m$

$$= \sum_{m=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - m\right) \sin(m\omega)$$

$$\therefore H(\omega) = \sum_{n=1}^{\frac{N-1}{2}} c(n) \sin(\omega n)$$

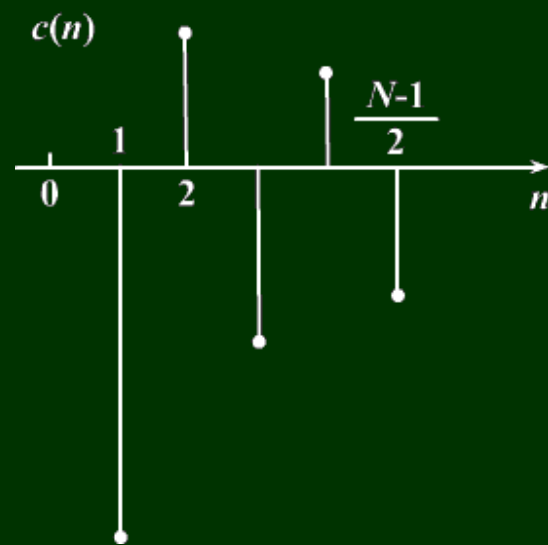
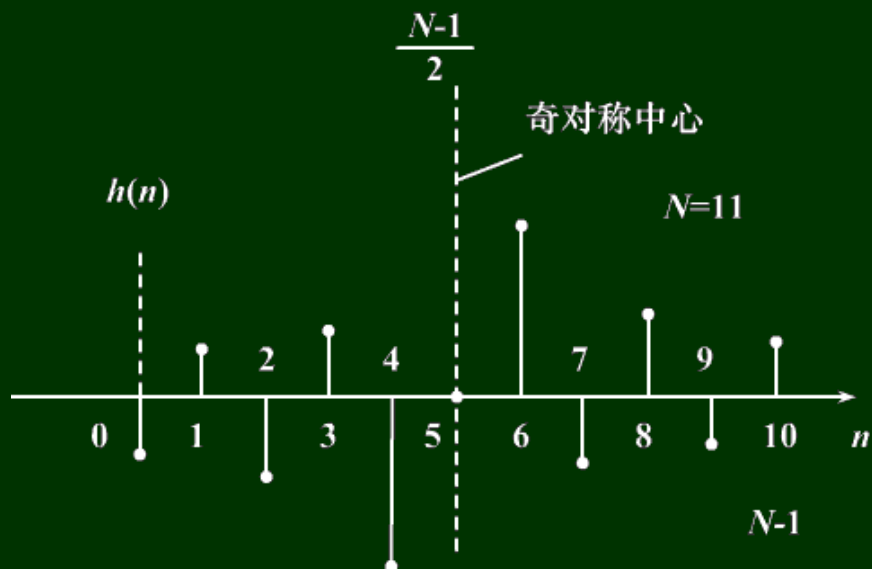
其中： $c(n) = 2h\left(\frac{N-1}{2} - n\right) \quad n = 1, \dots, \frac{N-1}{2}$




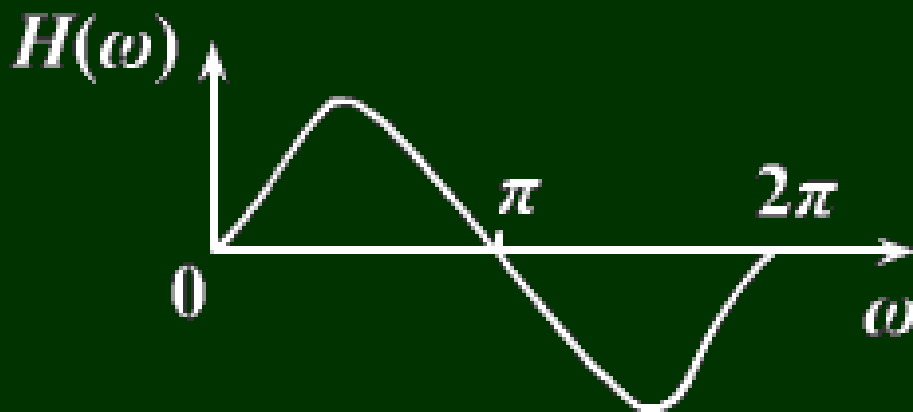


$$H(\omega) = \sum_{n=1}^{\frac{N-1}{2}} c(n) \sin(\omega n)$$

其中： $c(n) = 2h\left(\frac{N-1}{2} - n\right)$   $n = 1, \dots, \frac{N-1}{2}$




$$H(\omega) = \sum_{n=1}^{N-1} c(n) \sin(\omega n)$$



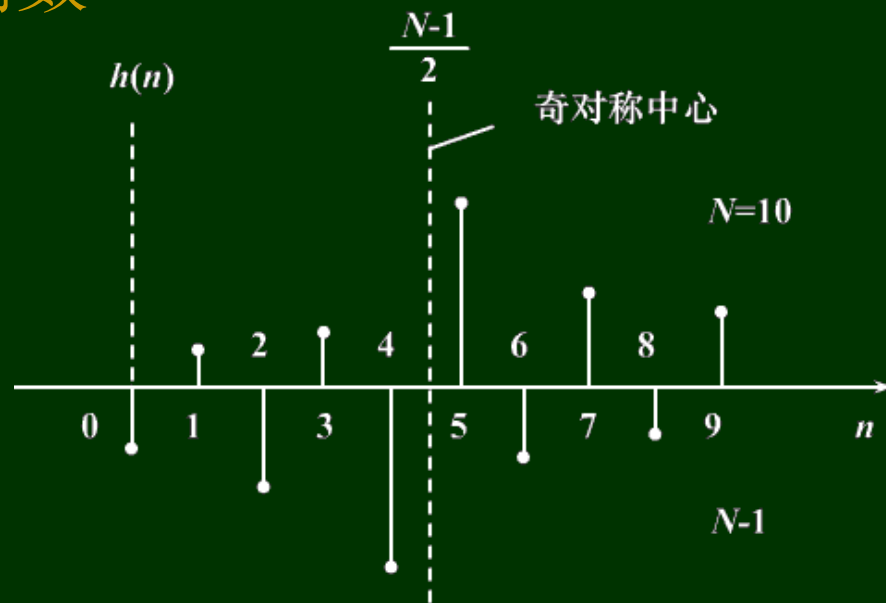
◆  $\omega = 0, \pi, 2\pi$  时  $\sin(\omega n) = 0$

则  $H(\omega) = 0 \quad \therefore z = \pm 1$  是零点

◆ 因  $\sin(\omega n)$  对  $\omega = 0, \pi, 2\pi$  呈奇对称

故  $H(\omega)$  对  $\omega = 0, \pi, 2\pi$  呈奇对称

#### 4) $h(n)$ 奇对称, $N$ 为偶数



幅度函数:

$$H(\omega) = \sum_{n=0}^{N-1} h(n) \sin \left[ \left( \frac{N-1}{2} - n \right) \omega \right]$$

$$= \sum_{n=0}^{\frac{N}{2}-1} 2h(n) \sin \left[ \left( \frac{N-1}{2} - n \right) \omega \right]$$





$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} 2h(n) \sin \left[ \left( \frac{N-1}{2} - n \right) \omega \right]$$

$$\text{令 } \frac{N}{2} - n = m$$

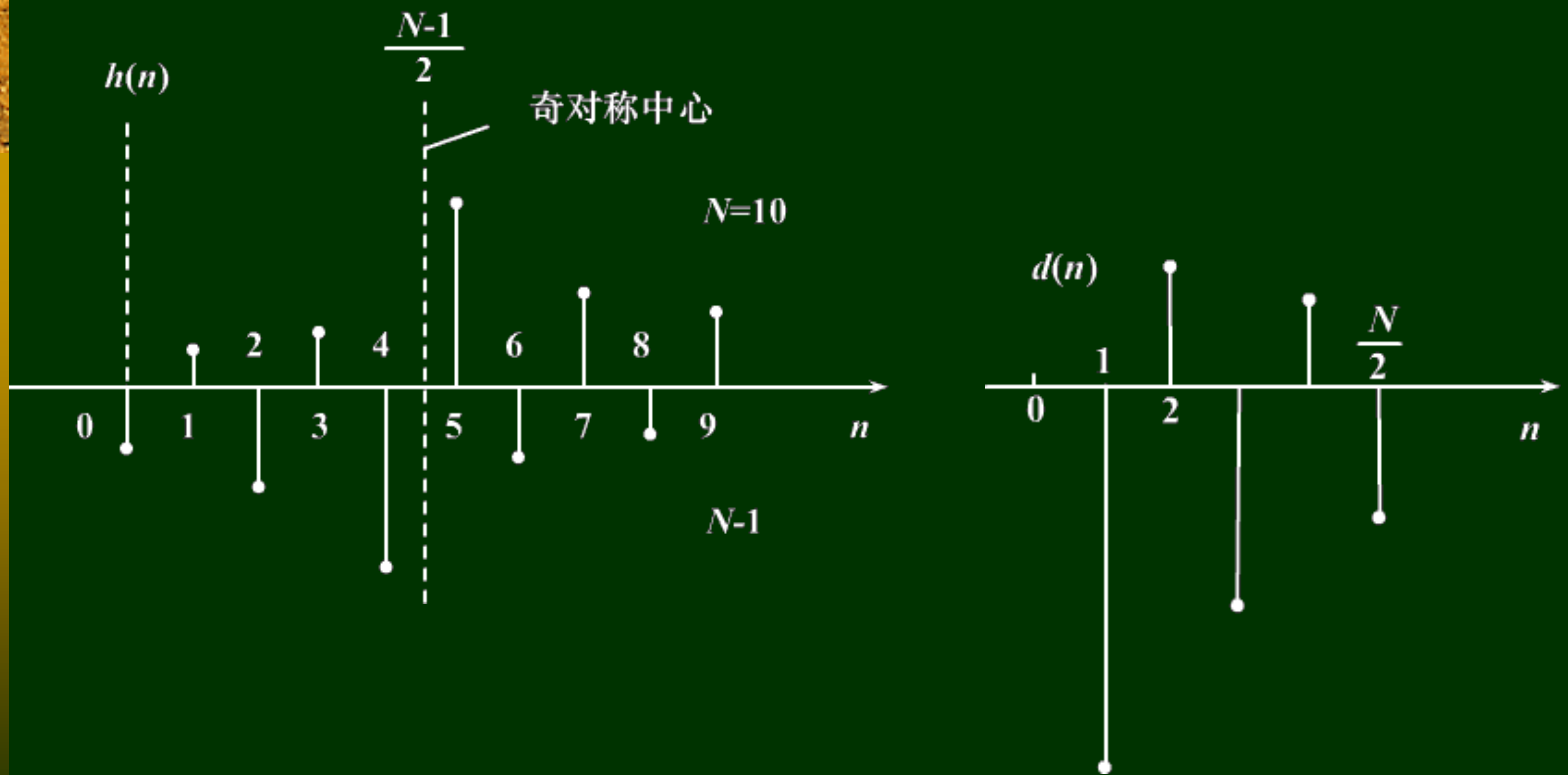
$$= \sum_{m=1}^{\frac{N}{2}} 2h \left( \frac{N}{2} - m \right) \sin \left[ \left( m - \frac{1}{2} \right) \omega \right]$$

$$\therefore H(\omega) = \sum_{n=1}^{N/2} d(n) \sin \left[ \omega \left( n - \frac{1}{2} \right) \right]$$

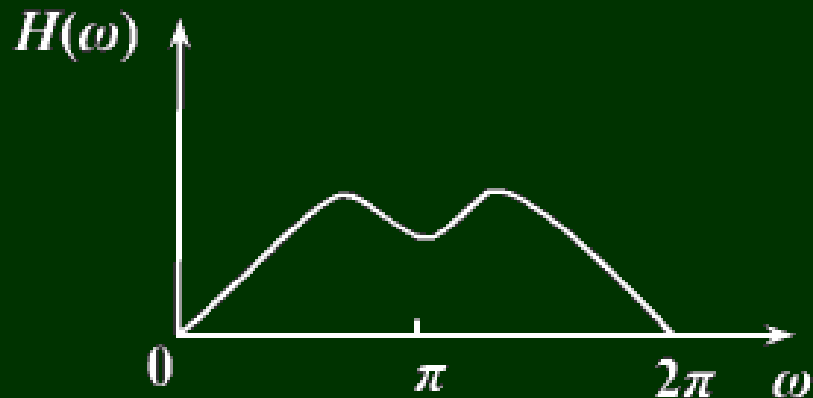
$$\text{其中: } d(n) = 2h \left( \frac{N}{2} - n \right) \quad n = 1, \dots, \frac{N}{2}$$

$$H(\omega) = \sum_{n=1}^{N/2} d(n) \sin \left[ \omega \left( n - \frac{1}{2} \right) \right]$$

其中：
$$d(n) = 2h \left( \frac{N}{2} - n \right) \quad n = 1, \dots, \frac{N}{2}$$



$$H(\omega) = \sum_{n=1}^{N/2} d(n) \sin \left[ \omega \left( n - \frac{1}{2} \right) \right]$$



◆  $\omega = 0, 2\pi$  时  $\sin \left[ \omega \left( n - \frac{1}{2} \right) \right] = 0$

则  $H(\omega) = 0 \quad \therefore z = 1$  是零点

◆  $H(\omega)$  对  $\omega = 0, 2\pi$  呈奇对称  $H(\omega)$  对  $\omega = \pi$  呈偶对称

◆  $h(n)$  为奇对称时, 有  $90^\circ$  相移, 适用于微分器和  $90^\circ$  移相器, 而选频滤波器采用  $h(n)$  为偶对称时

## 4、零点位置

由  $H(z) = \pm z^{-(N-1)} H(z^{-1})$  得:

1) 若  $z = z_i$  是  $H(z)$  的零点, 则  $z = z_i^{-1}$  也是零点

$$\because H(z_i) = 0$$

$$\therefore H(z_i^{-1}) = \pm z_i^{(N-1)} H(z_i) = 0$$

2)  $h(n)$  为实数, 则零点共轭成对

即  $z_i^*$ ,  $1/z_i^*$  也是零点

线性相位滤波器的零点是互为倒数的共轭对  
即共轭成对且镜像成对



$$1) \quad z_i = r_i e^{j\theta_i} \quad r_i \neq 1 \quad \theta_i \neq 0 \text{ 或 } \pi$$

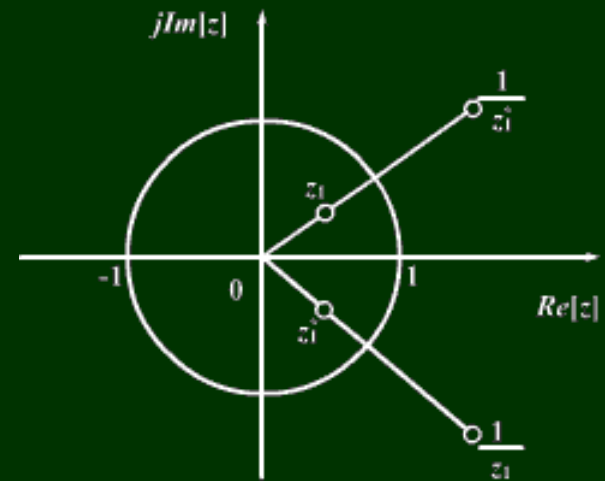
$$\text{零点: } r_i e^{j\theta_i} \quad r_i e^{-j\theta_i} \quad \frac{1}{r_i} e^{j\theta_i} \quad \frac{1}{r_i} e^{-j\theta_i}$$

$$H_i(z) = (1 - r_i e^{j\theta_i} z^{-1})(1 - r_i e^{-j\theta_i} z^{-1})$$

$$\cdot \left(1 - \frac{1}{r_i} e^{j\theta_i} z^{-1}\right) \left(1 - \frac{1}{r_i} e^{-j\theta_i} z^{-1}\right)$$

$$= \frac{1}{r_i^2} \left[1 - 2r_i \cos \theta_i z^{-1} + r_i^2 z^{-2}\right] \cdot \left[r_i^2 - 2r_i \cos \theta_i z^{-1} + z^{-2}\right]$$

$$N = 5 \quad \tau = \frac{N-1}{2} = 2$$





$$H(z_i) = \frac{1}{r_i^2} \left[ 1 - 2r_i \cos \theta_i z^{-1} + r_i^2 z^{-2} \right] \cdot \left[ r_i^2 - 2r_i \cos \theta_i z^{-1} + z^{-2} \right]$$

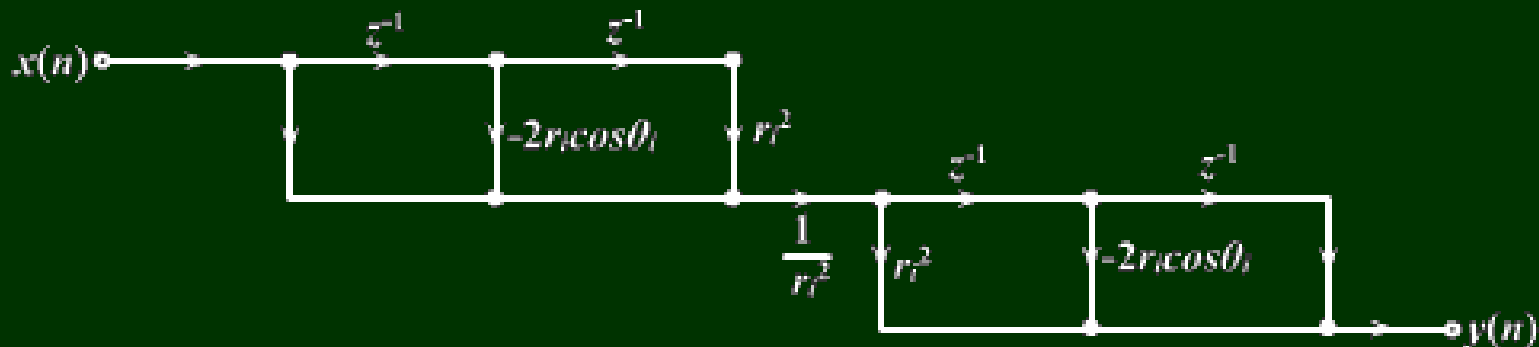


图7-6 线性相位FIR滤波器的级联结构

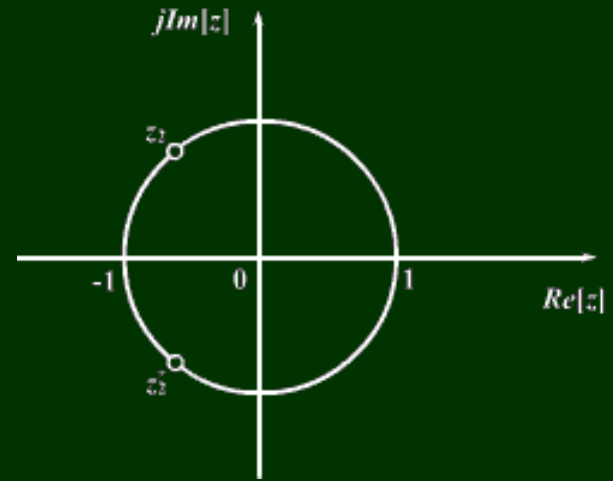
2)  $z_i = r_i e^{j\theta_i}$   $r_i = 1$   $\theta_i \neq 0$  或  $\pi$  , 即零点在单位圆上

零点:  $e^{j\theta_i}$   $e^{-j\theta_i}$

$$H_i(z) = (1 - e^{j\theta_i} z^{-1})(1 - e^{-j\theta_i} z^{-1})$$

$$= 1 - 2r \cos \theta_i z^{-1} + z^{-2}$$

$$N = 3 \quad \tau = \frac{N-1}{2} = 1$$



3)  $z_i = r_i e^{j\theta_i}$   $r_i \neq 1$   $\theta_i = 0$  或  $\pi$  , 即零点在实轴上

零点:  $r_i$   $\frac{1}{r_i}$

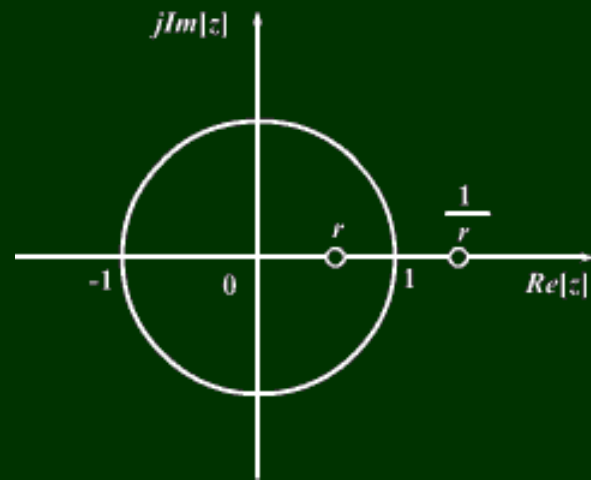
$$H_i(z) = (1 \pm r_i z^{-1}) \left( 1 \pm \frac{1}{r_i} z^{-1} \right)$$

$$= 1 \pm \left( r_i + \frac{1}{r_i} \right) z^{-1} + z^{-2}$$

"+"  $\theta_i = \pi$  负实轴上

"-"  $\theta_i = 0$  正实轴上

$$N = 3 \quad \tau = \frac{N-1}{2} = 1$$





$$4) z_i = r_i e^{j\theta_i} \quad r_i = 1 \quad \theta_i = 0 \text{ 或 } \pi$$

即零点既在实轴上，又在单位圆上

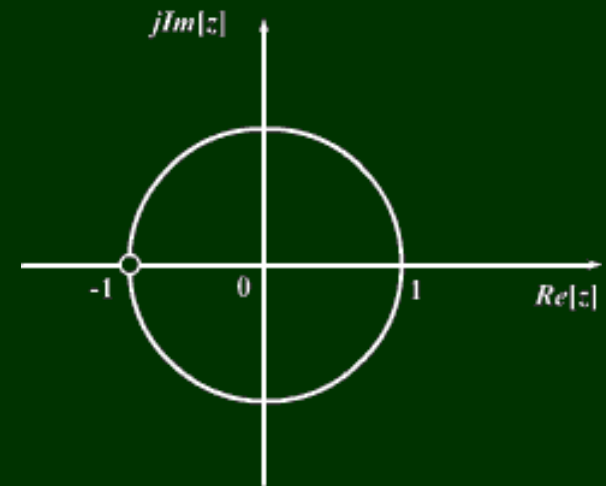
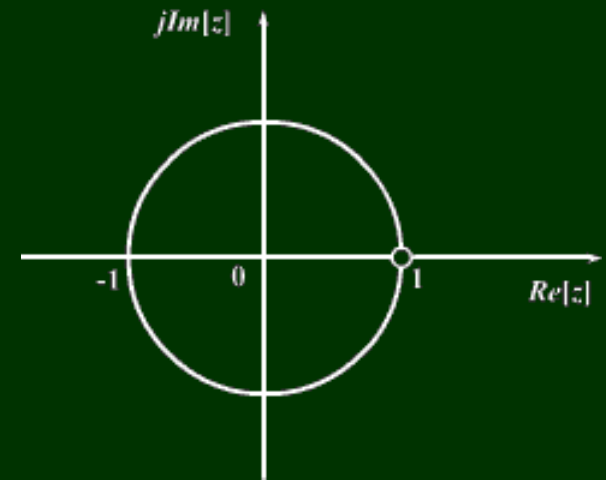
零点：  $\pm 1$

$$H_i(z) = (1 \pm z^{-1})$$

$$\text{"+"} \quad \theta_i = \pi \quad z = -1$$

$$\text{"-"} \quad \theta_i = 0 \quad z = 1$$

$$N = 2 \quad \tau = \frac{N-1}{2} = \frac{1}{2}$$



## 二、窗函数设计法

### 1、设计方法

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n} \rightarrow H_d(e^{j\omega})$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$h(n) = w(n)h_d(n)$$

$w(n)$ : 窗函数序列

要选择合适的形状和长度



以低通滤波器为例讨论：

线性相位理想低通滤波器的频率响应：

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & -\omega_c \leq \omega \leq \omega_c \\ 0 & -\pi \leq \omega \leq -\omega_c, \omega_c \leq \omega \leq \pi \end{cases}$$

其理想单位抽样响应：

$$h_d(n) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha} e^{j\omega n} d\omega = \frac{\omega_c}{\pi} \frac{\sin[\omega_c(n-\alpha)]}{\omega_c(n-\alpha)}$$

中心点为  $\alpha$  的偶对称无限长非因果序列

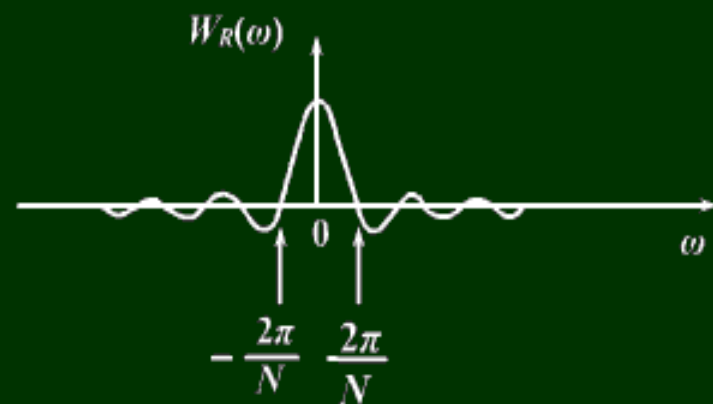
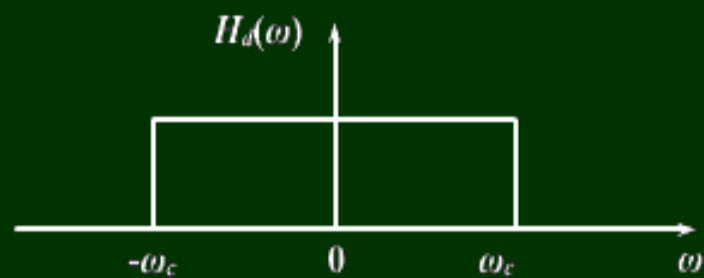
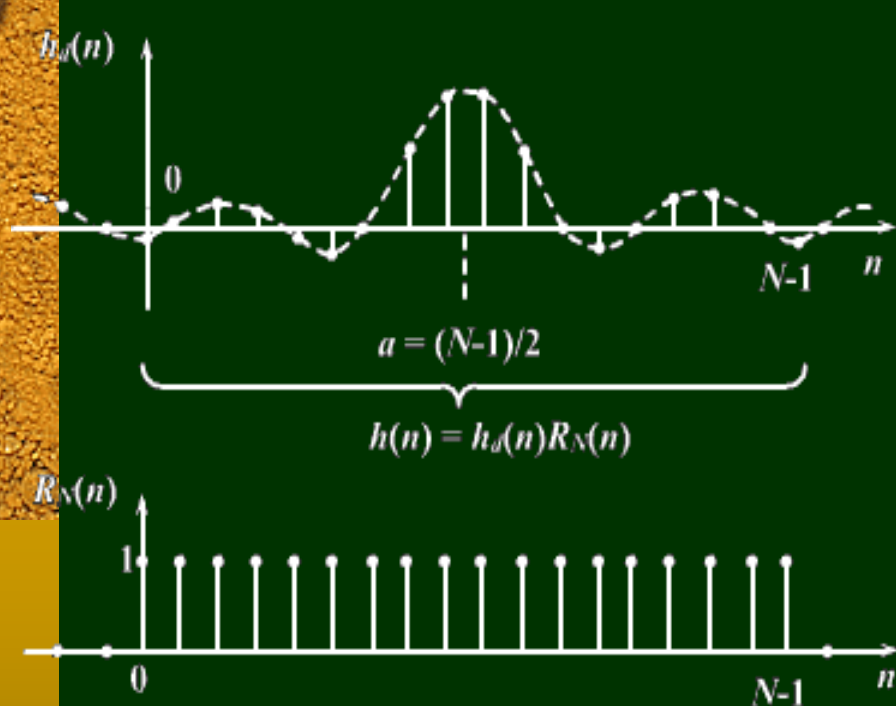
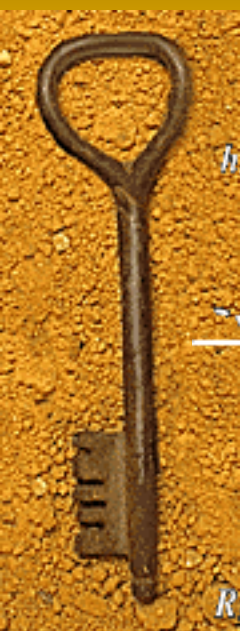


图7-7 理想矩形幅频特性的 $h_d(n)$ 和 $H_d(\omega)$ 以及矩形窗函数序列的 $w(n)=R_N(n)W_R(\omega)$



$$h_d(n) = \frac{\omega_c}{\pi} \frac{\sin[\omega_c(n - \alpha)]}{\omega_c(n - \alpha)}$$

取矩形窗： $w(n) = R_N(n)$

则FIR滤波器的单位抽样响应：

$$h(n) = h_d(n)w(n) = \begin{cases} h_d(n) & 0 \leq n \leq N-1 \\ 0 & \text{其它}n \end{cases}$$

按第一类线性相位条件，得  $\alpha = \frac{N-1}{2}$

$$\therefore h(n) = \begin{cases} \frac{\omega_c}{\pi} \frac{\sin\left[\omega_c\left(n - \frac{N-1}{2}\right)\right]}{\omega_c\left(n - \frac{N-1}{2}\right)} & 0 \leq n \leq N-1 \\ 0 & \text{其它}n \end{cases}$$

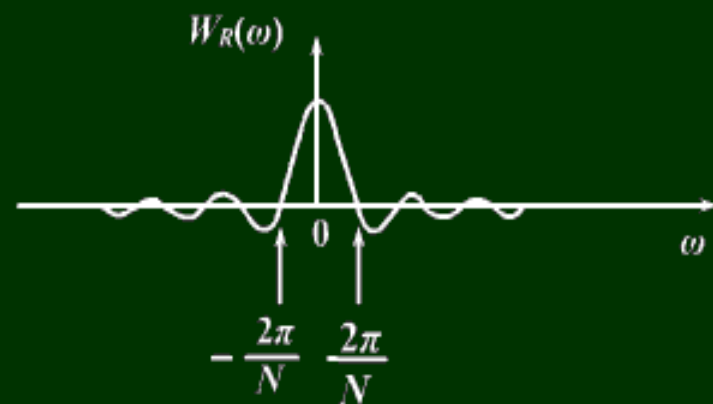
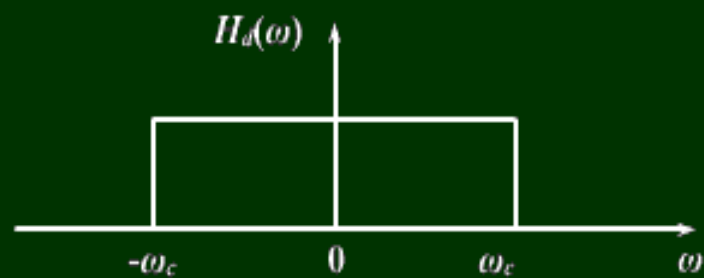
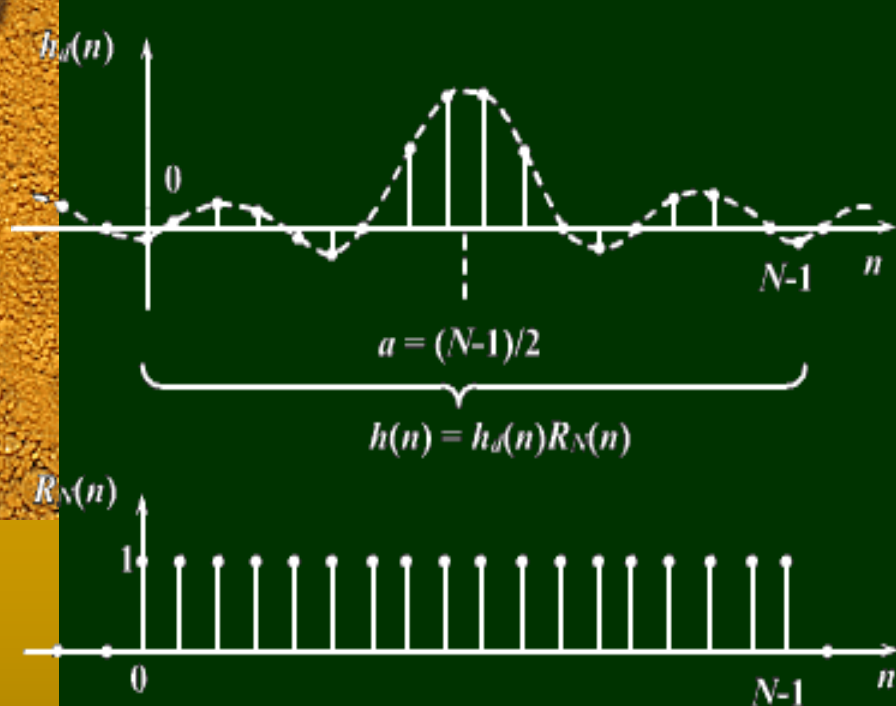
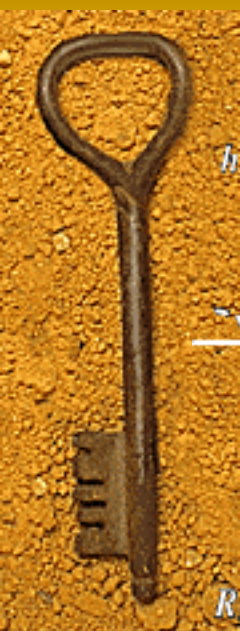


图7-7 理想矩形幅频特性的 $h_d(n)$ 和 $H_d(\omega)$ 以及矩形窗函数序列的 $w(n)=R_N(n)W_R(\omega)$

加窗处理后对频率响应的影响:

时域乘积相当于频域卷积  $h(n) = h_d(n)w(n)$

$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\theta}) W(e^{j(\omega-\theta)}) d\theta$$

而矩形窗的频率响应:

$$W_R(e^{j\omega}) = \sum_{n=0}^{N-1} w(n)e^{-j\omega n} = e^{-j\omega \frac{N-1}{2}} \frac{\sin \frac{\omega N}{2}}{\sin \frac{\omega}{2}}$$

其幅度函数:  $W_R(\omega) = \frac{\sin \frac{\omega N}{2}}{\sin \frac{\omega}{2}}$





理想滤波器的频率响应:

$$H_d(e^{j\omega}) = H_d(\omega)e^{-j\frac{N-1}{2}\omega}$$

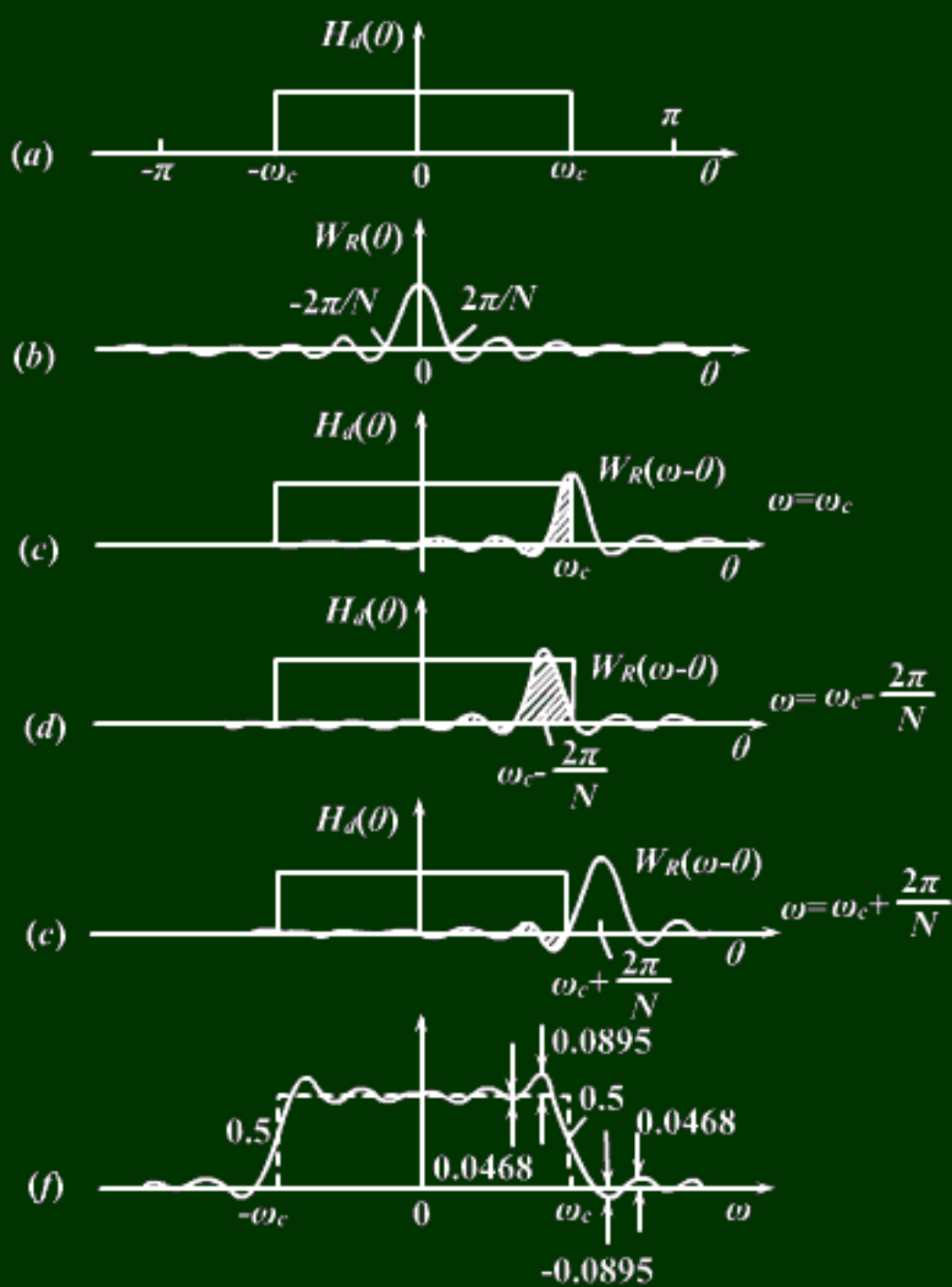
$$\text{其幅度函数: } H_d(\omega) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \omega_c \leq |\omega| \leq \pi \end{cases}$$


则FIR滤波器的频率响应:

$$\begin{aligned} H(e^{j\omega}) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta) e^{-j\frac{N-1}{2}\theta} W_R(\omega - \theta) e^{-j\frac{N-1}{2}(\omega - \theta)} d\theta \\ &= e^{-j\frac{N-1}{2}\omega} \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta) W_R(\omega - \theta) d\theta \end{aligned}$$

$$\text{其幅度函数: } H(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta) W_R(\omega - \theta) d\theta$$







幅度函数: 
$$H(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta) W_R(\omega - \theta) d\theta$$

- ◆  $\omega = 0$        $H(0)$ 近似于 $W_R(\theta)$ 的全部积分面积
- ◆  $\omega = \omega_c$        $H(\omega_c) = 0.5H(0)$
- ◆  $\omega = \omega_c - \frac{2\pi}{N}$        $H\left(\omega_c - \frac{2\pi}{N}\right)$ 为最大值, 正肩峰
- ◆  $\omega = \omega_c + \frac{2\pi}{N}$        $H\left(\omega_c + \frac{2\pi}{N}\right)$ 为最小值, 负肩峰
- ◆  $\omega > \omega_c + \frac{2\pi}{N}$       随 $\omega \uparrow$ ,  $H(\omega)$ 绕零值波动
- ◆  $\omega < \omega_c - \frac{2\pi}{N}$       随 $\omega \downarrow$ ,  $H(\omega)$ 绕 $H(0)$ 波动

## 加窗函数的影响:

- ◆ 不连续点处边沿加宽形成过渡带，其宽度（两肩峰之间的宽度）等于窗函数频率响应的主瓣宽度。
- ◆ 在  $\omega = \omega_c \pm \frac{2\pi}{N}$  处出现肩峰值，两侧形成起伏振荡，振荡的幅度和多少取决于旁瓣的幅度和多少
- ◆ 改变  $N$  只能改变窗谱的主瓣宽度，但不能改变主瓣与旁瓣的相对比例。其相对比例由窗函数形状决定，称为Gibbs效应

$$\text{幅度函数: } W_R(\omega) = \frac{\sin \frac{\omega N}{2}}{\sin \frac{\omega}{2}} \approx N \frac{\sin \frac{\omega N}{2}}{N \frac{\omega}{2}} = N \frac{\sin x}{x}$$



## 2、各种窗函数

### ◆ 窗函数的要求：

- 窗谱主瓣尽可能窄以获得较陡的过渡带
- 尽量减少窗谱最大旁瓣的相对幅度以减小肩峰和波纹



## ◆ 矩形窗

$$w(n) = R_N(n)$$

窗谱:

$$W_R(e^{j\omega}) = \sum_{n=0}^{N-1} w(n)e^{-j\omega n} = W_R(\omega)e^{-j\omega\frac{N-1}{2}}$$

幅度函数:

$$W_R(\omega) = \frac{\sin \frac{\omega N}{2}}{\sin \frac{\omega}{2}}$$

主瓣宽度最窄:  $\frac{4\pi}{N}$

旁瓣幅度大

◆ 三角形 (Bartlett) 窗

$$w(n) = \begin{cases} \frac{2n}{N-1} & 0 \leq n \leq \frac{N-1}{2} \\ 2 - \frac{2n}{N-1} & \frac{N-1}{2} \leq n \leq N-1 \end{cases}$$

窗谱:  $W(e^{j\omega}) = W(\omega) e^{-j\omega \frac{N-1}{2}}$

幅度函数:

$$W(\omega) = \frac{2}{N} \left[ \frac{\sin \frac{\omega N}{4}}{\sin \frac{\omega}{2}} \right]^2 \quad N \gg 1$$

主瓣宽度宽:  $\frac{8\pi}{N}$       旁瓣幅度较小





◆ 汉宁 (Hanning) 窗  
(升余弦窗)

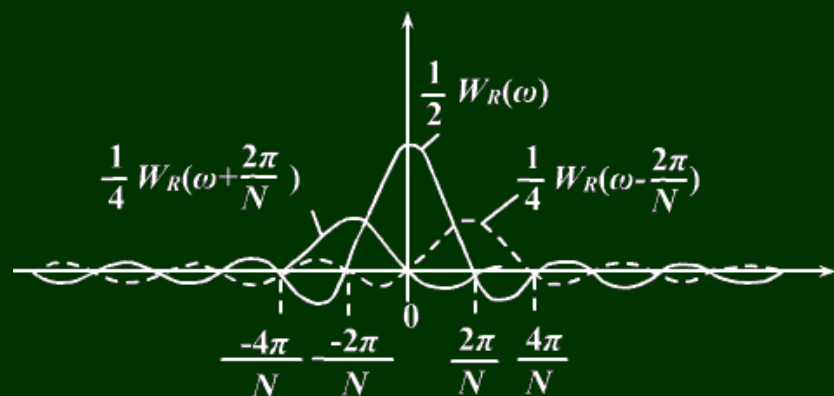
$$w(n) = \frac{1}{2} \left[ 1 - \cos \frac{2\pi n}{N-1} \right] R_N(n)$$

幅度函数: ( $N \gg 1$ )

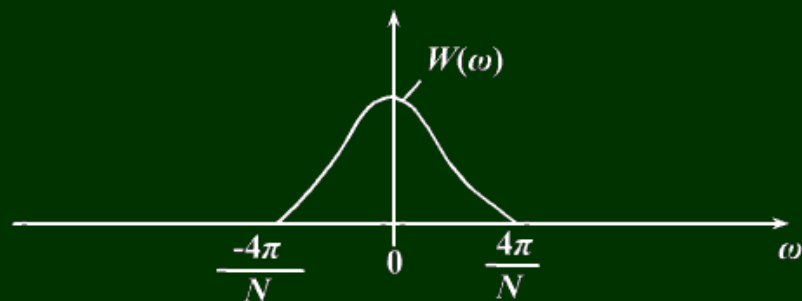
$$W(\omega) = 0.5W_R(\omega) + 0.25 \left[ W_R\left(\omega - \frac{2\pi}{N}\right) + W_R\left(\omega + \frac{2\pi}{N}\right) \right]$$

主瓣宽度宽:  $\frac{8\pi}{N}$

旁瓣幅度小



(a)



(b)

图7-9 汉宁(Hanning)窗谱



◆ 海明 (Hamming) 窗  
(改进的升余弦窗)

$$w(n) = \left[ 0.54 - 0.46 \cos \frac{2\pi n}{N-1} \right] R_N(n)$$

幅度函数: ( $N \gg 1$ )

$$W(\omega) = 0.54W_R(\omega) + 0.23 \left[ W_R\left(\omega - \frac{2\pi}{N}\right) + W_R\left(\omega + \frac{2\pi}{N}\right) \right]$$

主瓣宽度宽:  $\frac{8\pi}{N}$

旁瓣幅度更小



◆ 布莱克曼 (Blackman) 窗  
(二阶升余弦窗)

$$w(n) = \left[ 0.42 - 0.5 \cos \frac{2\pi n}{N-1} + 0.08 \cos \frac{4\pi n}{N-1} \right] R_N(n)$$

幅度函数: ( $N \gg 1$ )

$$W(\omega) = 0.42W_R(\omega) + 0.25 \left[ W_R\left(\omega - \frac{2\pi}{N}\right) + W_R\left(\omega + \frac{2\pi}{N}\right) \right] \\ + 0.04 \left[ W_R\left(\omega - \frac{4\pi}{N}\right) + W_R\left(\omega + \frac{4\pi}{N}\right) \right]$$

主瓣宽度最宽:  $\frac{12\pi}{N}$

旁瓣幅度最小

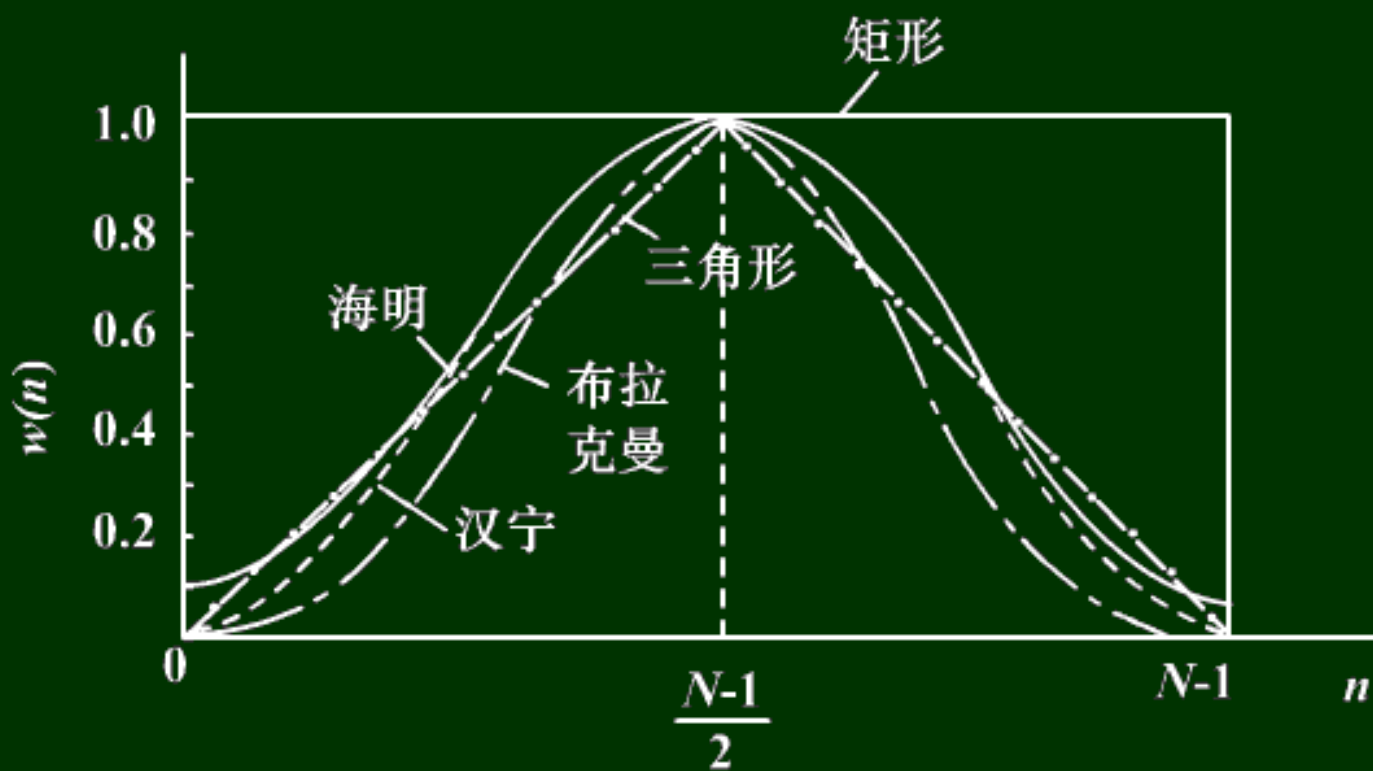
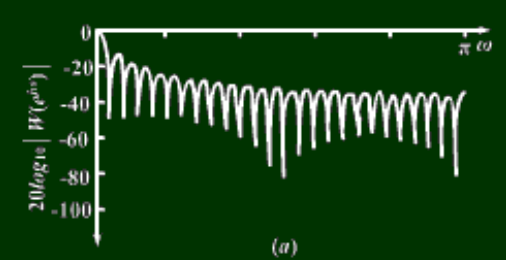
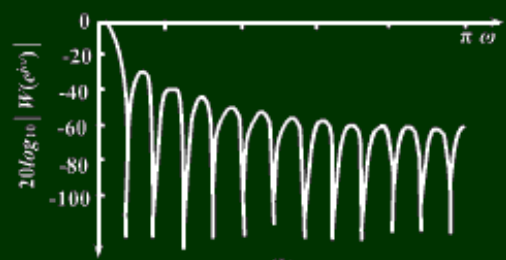


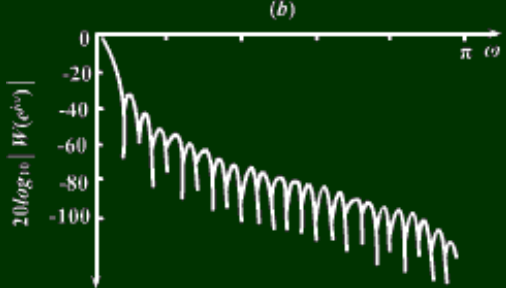
图7-10 设计有限长单位冲激响应滤波器常用的几种窗函数



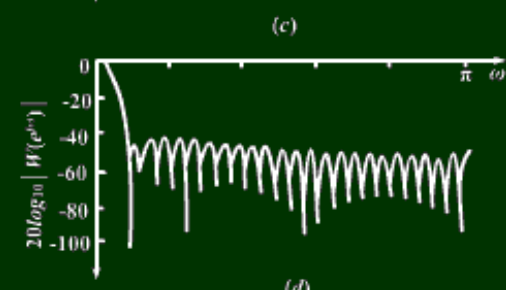
(a)



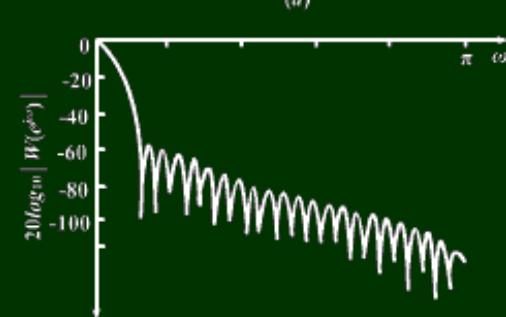
(b)



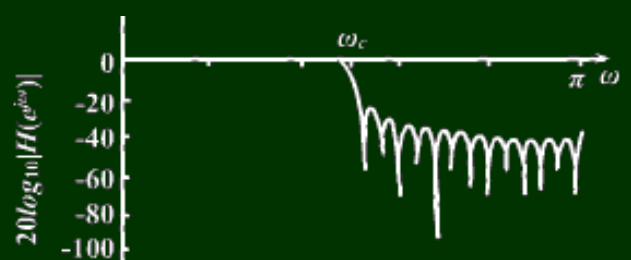
(c)



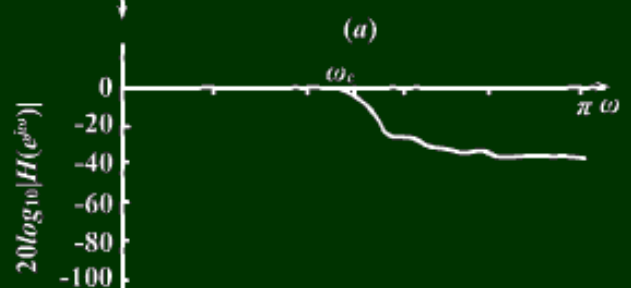
(d)



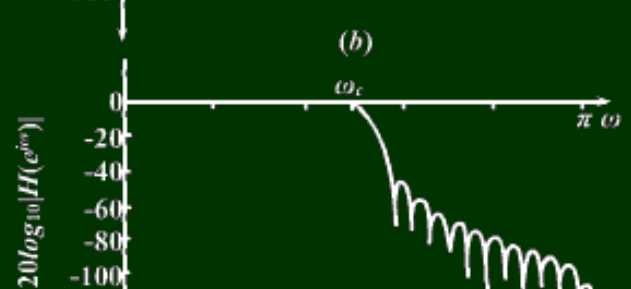
(e)



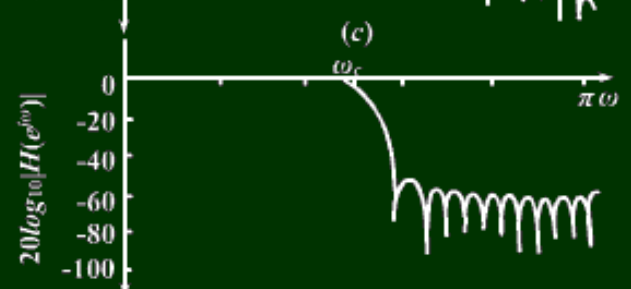
(a)



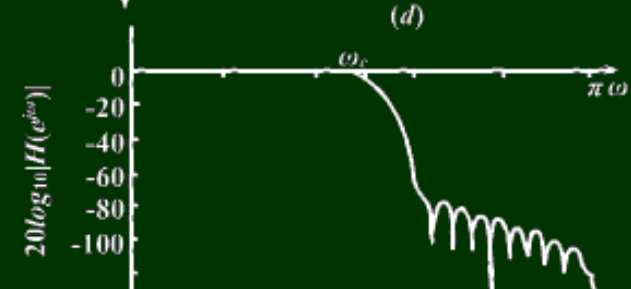
(b)



(c)



(d)



(e)

## ◆ 凯泽 (Kaiser) 窗

$$w(n) = \frac{I_0 \left( \beta \sqrt{1 - \left( 1 - \frac{2n}{N-1} \right)^2} \right)}{I_0(\beta)} \quad 0 \leq n \leq N-1$$

$I_0(\cdot)$ : 第一类变形零阶  
贝塞尔函数

改变  $\beta$  可同时调整主瓣  
宽度和旁瓣幅度

$\beta \uparrow$  旁瓣幅度  $\downarrow$   
但主瓣宽度  $\uparrow$

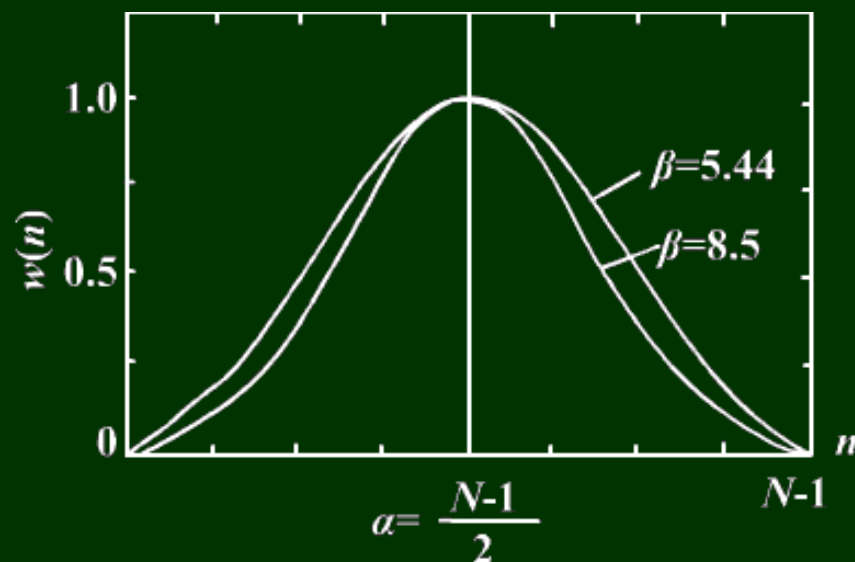


图7-13 凯泽窗函数

窗函数	窗谱性能指标		加窗后滤波器性能指标	
	旁瓣峰值 /dB	主瓣宽度 $/(2\pi/N)$	过渡带宽 $\Delta\omega$ $/(2\pi/N)$	阻带最小衰减 /dB
矩形窗	-13	2	0.9	-21
三角形窗	-25	4	2.1	-25
汉宁窗	-31	4	3.1	-44
海明窗	-41	4	3.3	-53
布拉克曼窗	-57	6	5.5	-74
凯泽窗 ( $\beta = 7.865$ )	-57		5	-80

阻带最小衰减只由窗形状决定

过渡带宽则与窗形状和窗宽 $N$ 都有关

### 3、窗函数法的设计步骤

- ◆ 给定理想的频率响应函数  $H_d(e^{j\omega})$  及技术指标  $\delta_2, \Delta\omega$
- ◆ 求出理想的单位抽样响应  $h_d(n)$
- ◆ 根据阻带衰减选择窗函数  $w(n)$
- ◆ 根据过渡带宽度确定  $N$  值  $N = A / \Delta\omega$
- ◆ 求所设计的FIR滤波器的单位抽样响应
$$h(n) = h_d(n) \cdot w(n)$$
- ◆ 计算频率响应  $H(e^{j\omega})$ ，验算指标是否满足要求




$$H_d(e^{j\omega}) \longrightarrow h_d(n)$$

公式法: 
$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

IFFT法:

对  $H_d(e^{j\omega})$  M点等间隔抽样:  $H_d(e^{j\frac{2\pi}{M}k})$

计算其IFFT, 得: 
$$h_M(n) = \sum_{r=-\infty}^{\infty} h_d(n + rM)$$

当  $M \gg N$  时,  $h_d(n) \approx h_M(n)$

## 4、线性相位FIR低通滤波器的设计

例：设计一个线性相位FIR低通滤波器，

给定抽样频率为  $\Omega_s = 2\pi \times 1.5 \times 10^4$  ( $rad/sec$ ),

通带截止频率为  $\Omega_p = 2\pi \times 1.5 \times 10^3$  ( $rad/sec$ ),

阻带起始频率为  $\Omega_{st} = 2\pi \times 3 \times 10^3$  ( $rad/sec$ ),

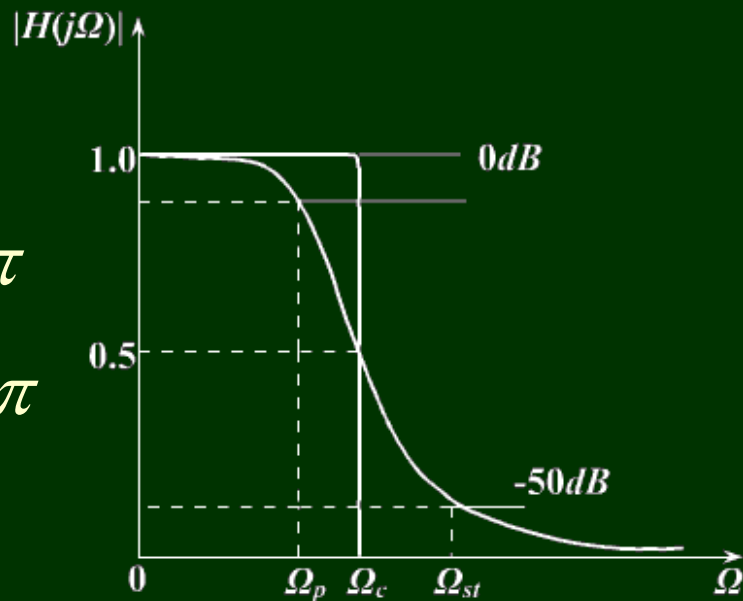
阻带衰减不小于  $-50dB$ ，幅度特性如图所示

解：1) 求数字频率

$$\omega_p = \Omega_p / f_s = 2\pi\Omega_p / \Omega_s = 0.2\pi$$

$$\omega_{st} = \Omega_{st} / f_s = 2\pi\Omega_{st} / \Omega_s = 0.4\pi$$

$$\delta_2 = 50dB$$





2) 求  $h_d(n)$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & -\omega_c \leq \omega \leq \omega_c \\ 0 & -\pi \leq \omega \leq -\omega_c, \omega_c \leq \omega \leq \pi \end{cases}$$

$$\omega_c = \frac{\Omega_c}{f_s} = 2\pi \frac{1/2(\Omega_p + \Omega_{st})}{\Omega_s} = 0.3\pi$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-j\omega\tau} e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega(n-\tau)} d\omega$$

$$= \begin{cases} \frac{1}{\pi(n-\tau)} \sin[\omega_c(n-\tau)] & n \neq \tau \\ \frac{\omega_c}{\pi} & n = \tau \end{cases} \quad \tau = \frac{N-1}{2}$$



3) 选择窗函数：由  $\delta_2 = 50dB$  确定海明窗（-53dB）

$$w(n) = \left[ 0.54 - 0.46 \cos \frac{2\pi n}{N-1} \right] R_N(n)$$

4) 确定  $N$  值      海明窗带宽：  $\Delta\omega = \frac{6.6\pi}{N}$

$$\Delta\omega = 2\pi \frac{\Omega_{st} - \Omega_p}{\Omega_s} = 0.2\pi$$

$$N = \frac{A}{\Delta\omega} = \frac{6.6\pi}{0.2\pi} = 33$$

$$\tau = \frac{N-1}{2} = 16$$

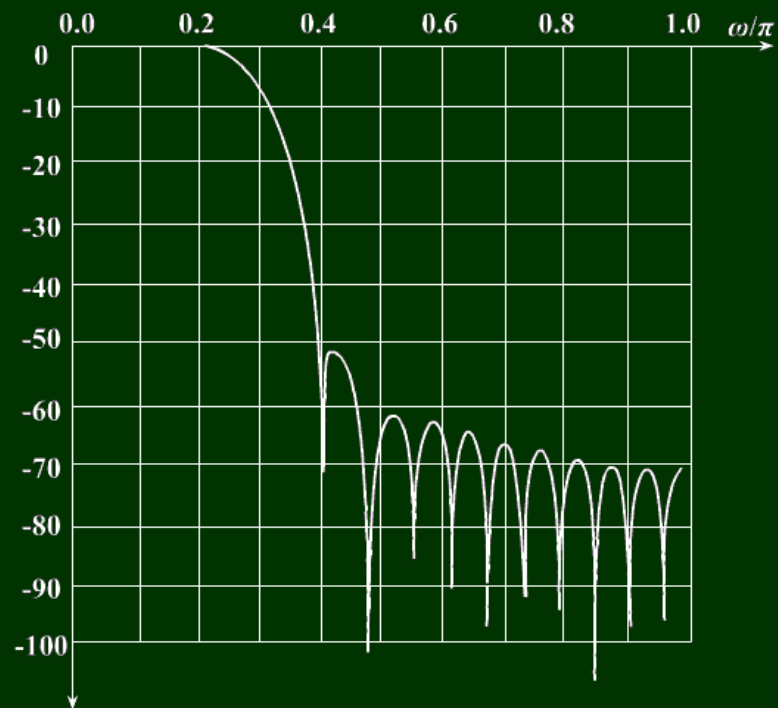
## 5) 确定FIR滤波器的 $h(n)$

$$h(n) = h_d(n)w(n)$$

$$= \frac{\sin\left[0.3\pi(n-16)\right]}{\pi(n-16)} \cdot \left[0.54 - 0.46\cos\frac{\pi n}{16}\right] R_{33}(n)$$

## 6) 求 $H(e^{j\omega})$ , 验证

若不满足, 则改变 $N$   
或窗形状重新设计



过度带宽 $\Delta\omega$ :  $0.3476563\pi$

第一通带波纹: 0.020837dB

第一阻带最小衰减: 50.9159dB

## 5、线性相位FIR高通滤波器的设计

理想高通的频响:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & \omega_c \leq |\omega| \leq \pi \\ 0 & \text{其它}\omega \end{cases} \quad \tau = \frac{N-1}{2}$$

其单位抽样响应:

$$h_d(n) = \frac{1}{2\pi} \left[ \int_{-\pi}^{-\omega_c} e^{j\omega(n-\tau)} d\omega + \int_{\omega_c}^{\pi} e^{j\omega(n-\tau)} d\omega \right]$$
$$= \begin{cases} \frac{1}{\pi(n-\tau)} \left\{ \sin[\pi(n-\tau)] - \sin[\omega_c(n-\tau)] \right\} & n \neq \tau \\ \frac{1}{\pi} (\pi - \omega_c) & n = \tau \end{cases}$$

高通滤波器( $\omega_c$ ) = 全通滤波器 - 低通滤波器( $\omega_c$ )

## 6、线性相位FIR带通滤波器的设计

理想带通的频响：

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & 0 < \omega_1 \leq |\omega| \leq \omega_2 < \pi \\ 0 & \text{其它 } \omega \end{cases}$$

$$\tau = \frac{N-1}{2}$$

其单位抽样响应：

$$h_d(n) = \frac{1}{2\pi} \left[ \int_{-\omega_2}^{-\omega_1} e^{j\omega(n-\tau)} d\omega + \int_{\omega_1}^{\omega_2} e^{j\omega(n-\tau)} d\omega \right]$$

$$= \begin{cases} \frac{1}{\pi(n-\tau)} \left\{ \sin[\omega_2(n-\tau)] - \sin[\omega_1(n-\tau)] \right\} & n \neq \tau \\ \frac{1}{\pi} (\omega_2 - \omega_1) & n = \tau \end{cases}$$

带通滤波器 $(\omega_1, \omega_2)$  = 低通滤波器 $(\omega_2)$  - 低通滤波器 $(\omega_1)$



## 7、线性相位FIR带阻滤波器的设计

理想带阻的频响：

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & 0 \leq |\omega| \leq \omega_1, \omega_2 \leq |\omega| \leq \pi \\ 0 & \text{其它 } \omega \end{cases}$$

$$\tau = \frac{N-1}{2}$$

其单位抽样响应：

$$h_d(n) = \frac{1}{2\pi} \left[ \int_{-\pi}^{-\omega_2} e^{j\omega(n-\tau)} d\omega + \int_{-\omega_1}^{\omega_1} e^{j\omega(n-\tau)} d\omega + \int_{\omega_2}^{\pi} e^{j\omega(n-\tau)} d\omega \right]$$

$$= \begin{cases} \frac{1}{\pi(n-\tau)} \left\{ \sin[\pi(n-\tau)] + \sin[\omega_1(n-\tau)] - \sin[\omega_2(n-\tau)] \right\} & n \neq \tau \\ \frac{1}{\pi} (\pi + \omega_1 - \omega_2) & n = \tau \end{cases}$$

带阻滤波器( $\omega_1, \omega_2$ ) = 高通滤波器( $\omega_2$ ) + 低通滤波器( $\omega_1$ )

# 三、频率抽样设计法

## 1、设计方法

对理想频率响应等间隔抽样

作为实际FIR数字滤波器的频率特性的抽样值

$$H(k) = H_d(k) = H_d(e^{j\omega}) \Big|_{\omega = \frac{2\pi}{N}k} \quad k = 0, 1, \dots, N-1$$

The diagram illustrates the relationships between the sampled frequency response  $H(k)$ , the impulse response  $h(n)$ , the z-transform  $H(z)$ , and the continuous frequency response  $H(e^{j\omega})$ . Arrows show that  $H(k)$  is derived from  $H_d(e^{j\omega})$  at discrete frequencies  $\omega = \frac{2\pi}{N}k$ , and  $H(k)$  is the discrete-time Fourier transform of  $h(n)$ .  $H(z)$  is the z-transform of  $h(n)$ , and  $H(e^{j\omega})$  is the frequency response of  $H(z)$ .



窗函数设计法:

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n} \rightarrow H_d(e^{j\omega})$$

↓

$$h(n) = w(n)h_d(n) \longrightarrow h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$



内插公式:

$$H(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - W_N^{-k} z^{-1}}$$

$$H(e^{j\omega}) = \sum_{k=0}^{N-1} H(k) \Phi\left(\omega - \frac{2\pi}{N}k\right)$$

$$\Phi(\omega) = \frac{1}{N} \frac{\sin \frac{\omega N}{2}}{\sin \frac{\omega}{2}} e^{-j\omega \frac{N-1}{2}}$$

$$H(e^{j\omega}) = e^{-j\frac{N-1}{2}\omega} \sum_{k=0}^{N-1} H(k) \frac{1}{N} e^{j\frac{\pi k}{N}(N-1)} \frac{\sin \left[ N \left( \frac{\omega}{2} - \frac{\pi k}{N} \right) \right]}{\sin \left( \frac{\omega}{2} - \frac{\pi k}{N} \right)}$$





- ◆ 抽样点上，频率响应严格相等
- ◆ 抽样点之间，加权内插函数的延伸叠加
- ◆ 变化越平缓，内插越接近理想值，逼近误差较小

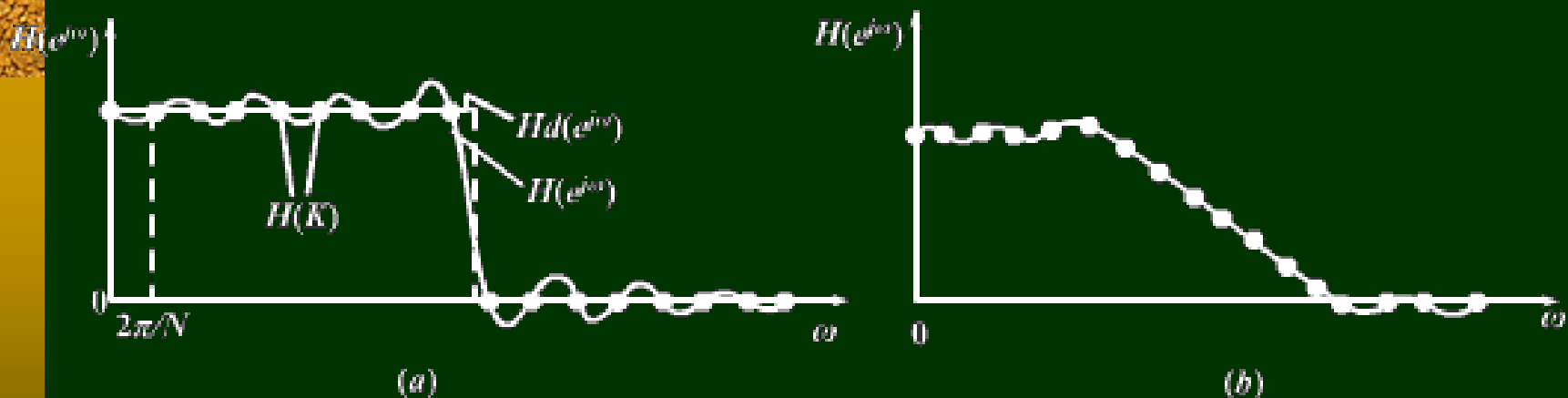


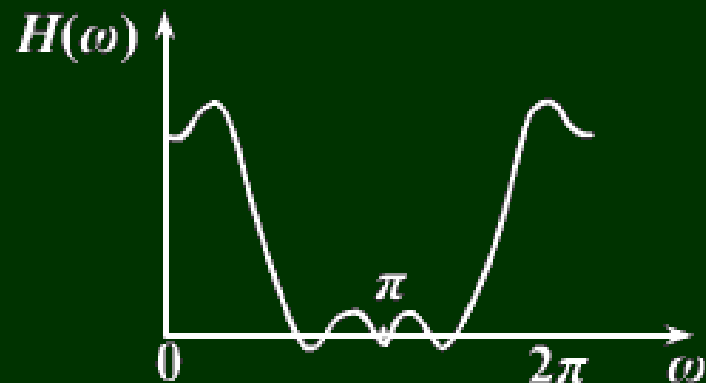
图7-16 频率抽样的响应

# 1、线性相位的约束

1)  $h(n)$ 偶对称,  $N$ 为奇数

$$H(e^{j\omega}) = H(\omega)e^{-j\frac{N-1}{2}\omega}$$

$$H(\omega) = H(2\pi - \omega)$$



$$H(k) = H\left(e^{j\frac{2\pi}{N}k}\right) = H_k e^{j\theta_k} \quad \omega = \frac{2\pi}{N}k$$

幅度偶对称:  $H_k = H_{N-k}$

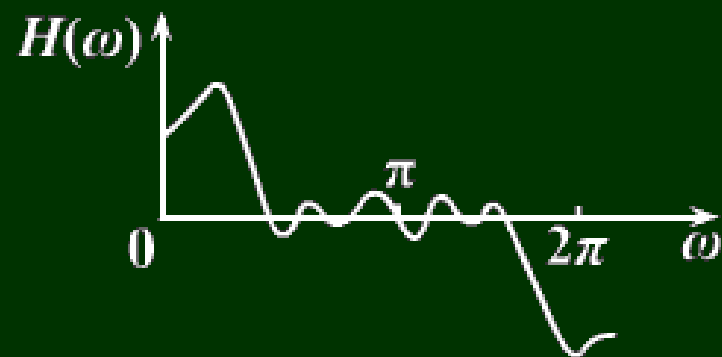
相位函数:  $\theta_k = -\frac{N-1}{2} \cdot \frac{2\pi}{N}k = -k\pi\left(1 - \frac{1}{N}\right)$

2)  $h(n)$ 偶对称,  $N$ 为偶数

$$H(e^{j\omega}) = H(\omega)e^{-j\frac{N-1}{2}\omega}$$

$$H(\omega) = -H(2\pi - \omega)$$

$$H(k) = H\left(e^{j\frac{2\pi}{N}k}\right) = H_k e^{j\theta_k} \quad \omega = \frac{2\pi}{N}k$$



幅度奇对称:  $H_k = -H_{N-k}$

相位函数:  $\theta_k = -k\pi\left(1 - \frac{1}{N}\right)$

## 2、频率抽样的两种方法

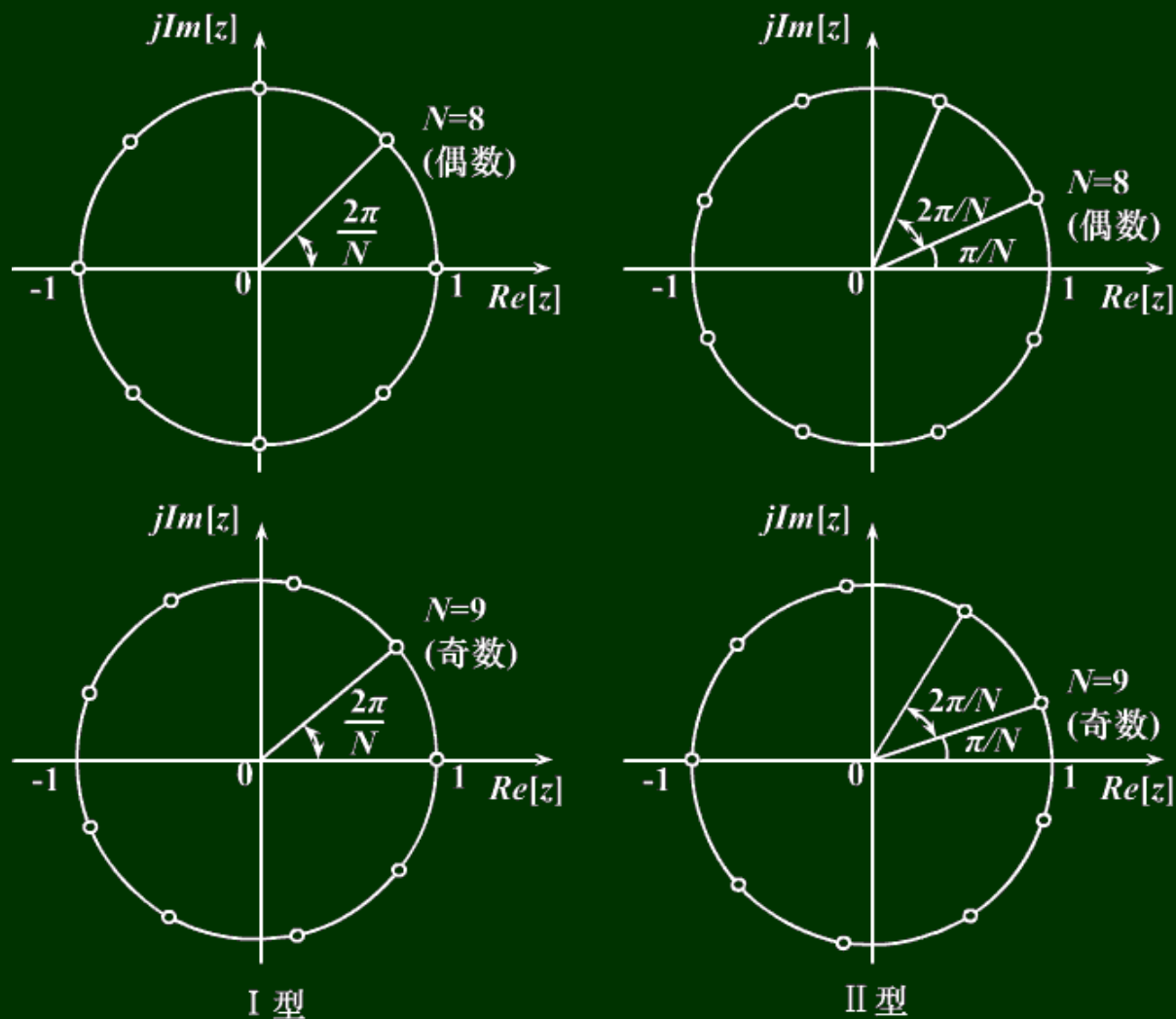
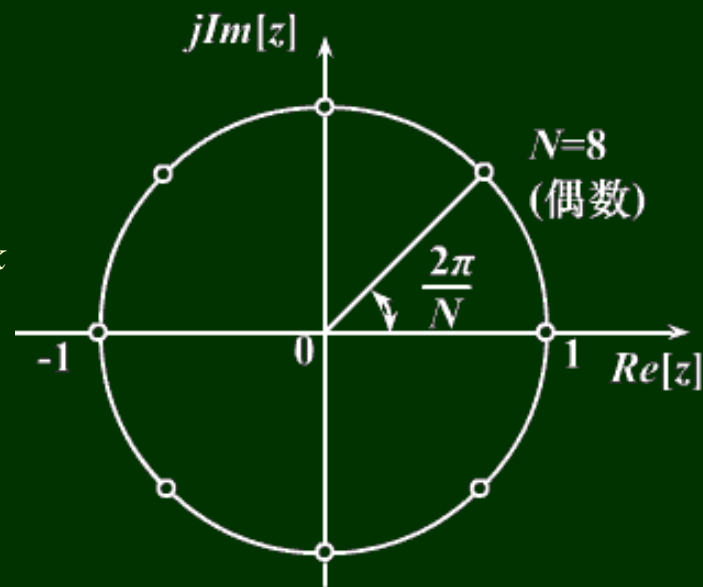


图7-17 两种频率抽样(I 型, II 型)

# 1) 第一种频率抽样

$$H(k) = H_d(k) = H_d(e^{j\omega}) \Big|_{\omega = \frac{2\pi}{N}k}$$

$$k = 0, 1, \dots, N-1$$



系统函数:

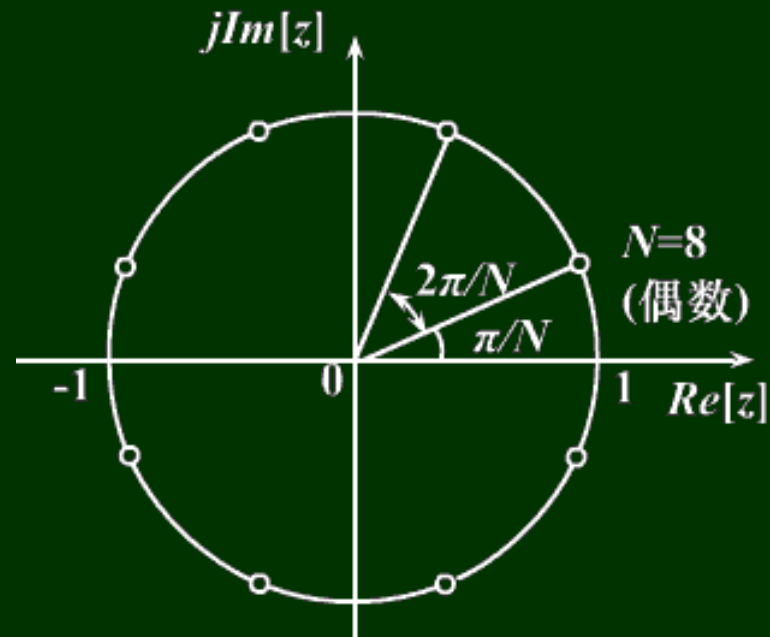
$$H(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - W_N^{-k} z^{-1}}$$

频率响应:

$$H(e^{j\omega}) = \frac{1}{N} e^{-j\frac{N-1}{2}\omega} \sum_{k=0}^{N-1} H(k) e^{-j\frac{\pi k}{N}} \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2} - \frac{\pi k}{N}\right)}$$

## 2) 第二种频率抽样

$$H(k) = H_d(e^{j\omega}) \Big|_{\omega = \frac{2\pi}{N}k + \frac{\pi}{N}}$$
$$k = 0, 1, \dots, N-1$$



系统函数:

$$H(z) = \frac{1 + z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - e^{j\frac{2\pi}{N}\left(k + \frac{1}{2}\right)} z^{-1}}$$

频率响应:

$$H(e^{j\omega}) = \frac{\cos\left(\frac{\omega N}{2}\right)}{N} e^{-j\frac{N-1}{2}\omega} \sum_{k=0}^{N-1} \frac{H(k) e^{-j\frac{\pi}{N}\left(k + \frac{1}{2}\right)}}{j \sin\left[\frac{\omega}{2} - \frac{\pi}{N}\left(k + \frac{1}{2}\right)\right]}$$

### 3、线性相位第一种频率抽样

$h(n)$ 为实数序列时， $H(k)$ 圆周共轭对称

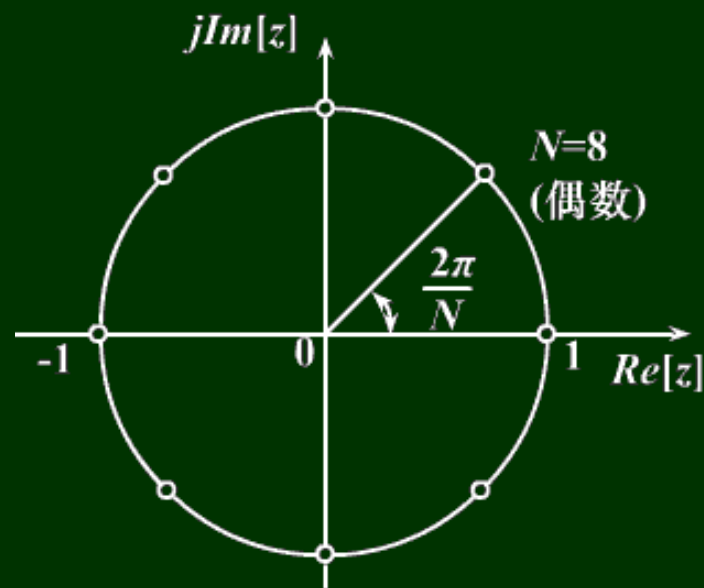
$$H(k) = H^*(N - k)$$

即：  $|H(k)| = |H(N - k)|$

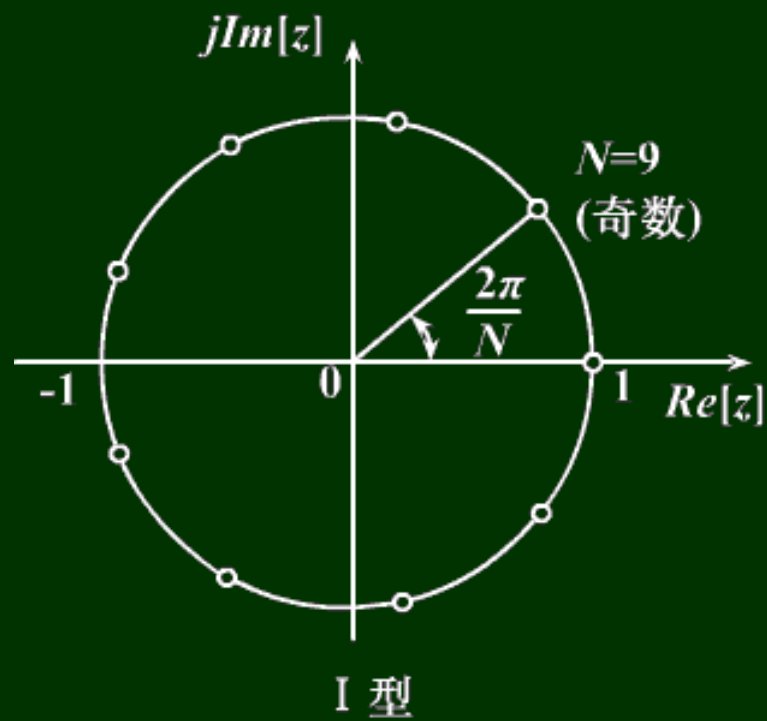
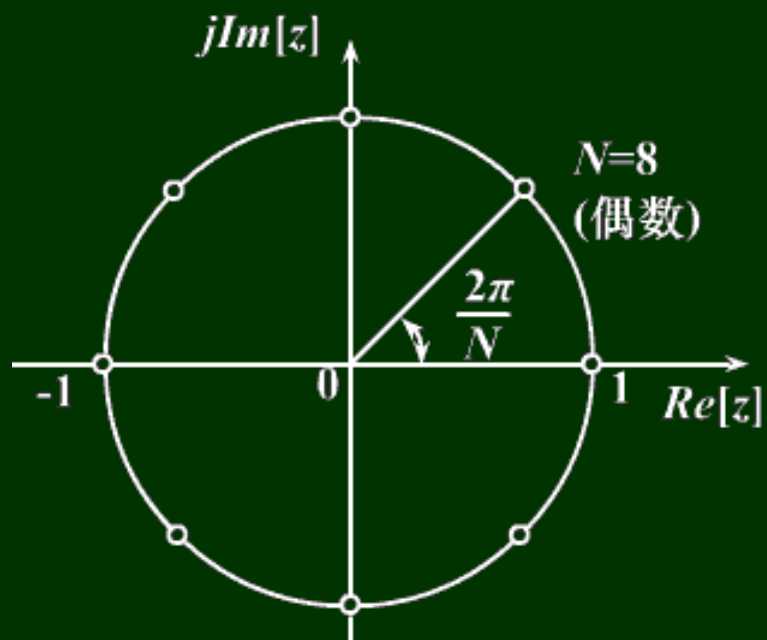
$$\theta(k) = -\theta(N - k)$$

对称中心：  $\frac{N}{2}$

又线性相位：  $\theta(e^{j\omega}) = -\frac{N-1}{2}\omega$







当 $N$ 为奇数时:

$$\theta(k) = \begin{cases} -\frac{2\pi}{N}k\left(\frac{N-1}{2}\right) & k = 0, \dots, \frac{N-1}{2} \\ \frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right) & k = \frac{N+1}{2}, \dots, N-1 \end{cases}$$

当 $N$ 为偶数时:

$$\theta(k) = \begin{cases} -\frac{2\pi}{N}k\left(\frac{N-1}{2}\right) & k = 0, \dots, \left(\frac{N}{2} - 1\right) \\ 0 & k = \frac{N}{2} \\ \frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right) & k = \left(\frac{N}{2} + 1\right), \dots, N-1 \end{cases}$$



当 $N$ 为奇数时:

$$H(k) = \begin{cases} |H(k)| e^{-j\frac{2\pi}{N}k\left(\frac{N-1}{2}\right)} & k = 0, \dots, \frac{N-1}{2} \\ |H(N-k)| e^{j\frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right)} & k = \frac{N+1}{2}, \dots, N-1 \end{cases}$$

当 $N$ 为偶数时:

$$H(k) = \begin{cases} |H(k)| e^{-j\frac{2\pi}{N}k\left(\frac{N-1}{2}\right)} & k = 0, \dots, \left(\frac{N}{2} - 1\right) \\ 0 & k = \frac{N}{2} \\ |H(N-k)| e^{j\frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right)} & k = \left(\frac{N}{2} + 1\right), \dots, N-1 \end{cases}$$



频率响应:

$$H(e^{j\omega}) = e^{-j\frac{N-1}{2}\omega} \left\{ \frac{|H(0)| \sin\left(\frac{\omega N}{2}\right)}{N \sin\left(\frac{\omega}{2}\right)} \right.$$

$$\left. + \sum_{k=1}^M \frac{|H(k)|}{N} \left[ \frac{\sin\left[N\left(\frac{\omega}{2} - \frac{k\pi}{N}\right)\right]}{\sin\left(\frac{\omega}{2} - \frac{k\pi}{N}\right)} + \frac{\sin\left[N\left(\frac{\omega}{2} + \frac{k\pi}{N}\right)\right]}{\sin\left(\frac{\omega}{2} + \frac{k\pi}{N}\right)} \right] \right\}$$

当 $N$ 为奇数时:  $M = \frac{N-1}{2}$

当 $N$ 为偶数时:  $M = \frac{N}{2} - 1$



## 4、线性相位第二种频率抽样

$h(n)$ 为实数序列时， $H(k)$ 圆周共轭对称

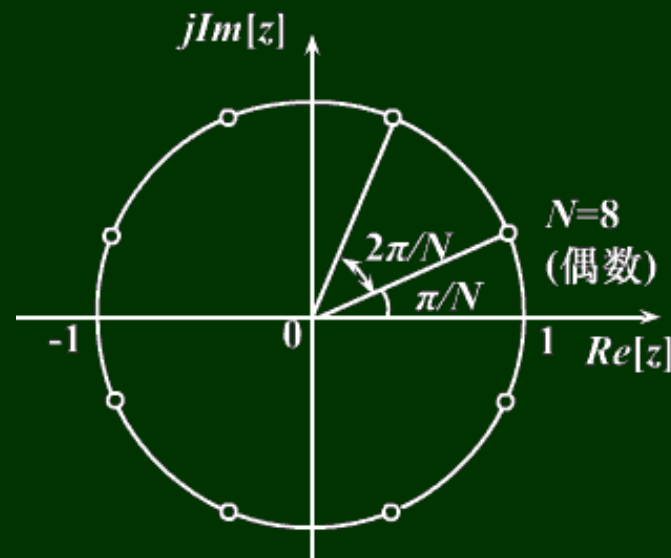
$$H(k) = H^*(N-1-k)$$

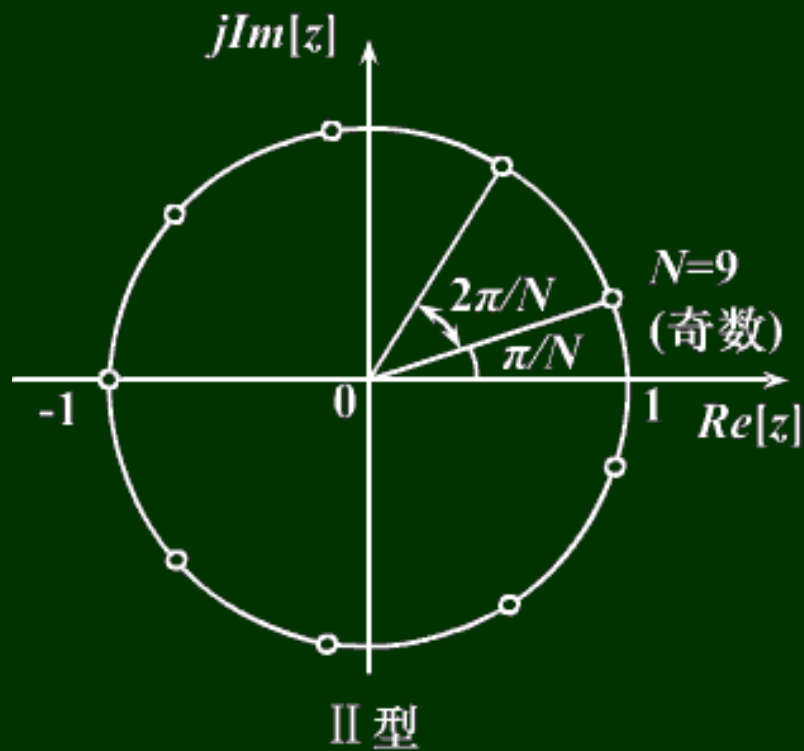
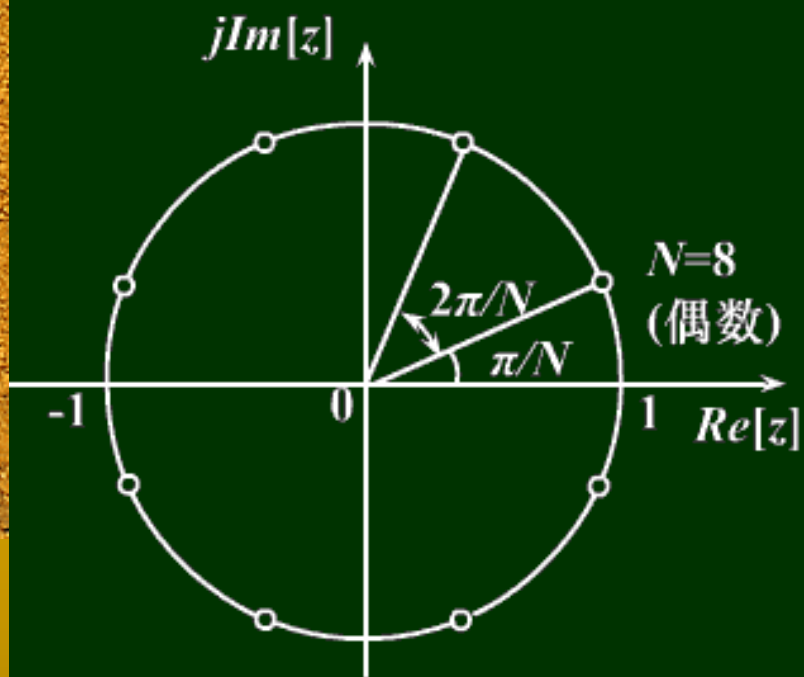
即：  $|H(k)| = |H(N-1-k)|$

$$\theta(k) = -\theta(N-1-k)$$

对称中心： $\frac{N-1}{2}$

又线性相位： $\theta(e^{j\omega}) = -\frac{N-1}{2}\omega$





当 $N$ 为奇数时:


$$\theta(k) = \begin{cases} -\frac{2\pi}{N} \left( k + \frac{1}{2} \right) \left( \frac{N-1}{2} \right) & k = 0, \dots, \frac{N-3}{2} \\ 0 & k = \frac{N-1}{2} \\ \frac{2\pi}{N} \left( N - k - \frac{1}{2} \right) \left( \frac{N-1}{2} \right) & k = \frac{N+1}{2}, \dots, N-1 \end{cases}$$

当 $N$ 为偶数时:

$$\theta(k) = \begin{cases} -\frac{2\pi}{N} \left( k + \frac{1}{2} \right) \left( \frac{N-1}{2} \right) & k = 0, \dots, \left( \frac{N}{2} - 1 \right) \\ \frac{2\pi}{N} \left( N - k - \frac{1}{2} \right) \left( \frac{N-1}{2} \right) & k = \frac{N}{2}, \dots, N-1 \end{cases}$$



当 $N$ 为奇数时:


$$H(k) = \begin{cases} |H(k)| e^{-j\frac{2\pi}{N}\left(k+\frac{1}{2}\right)\left(\frac{N-1}{2}\right)} & k = 0, \dots, \frac{N-3}{2} \\ \left| H\left(\frac{N-1}{2}\right) \right| & k = \frac{N-1}{2} \\ |H(N-1-k)| e^{j\frac{2\pi}{N}\left(N-k-\frac{1}{2}\right)\left(\frac{N-1}{2}\right)} & k = \frac{N+1}{2}, \dots, N-1 \end{cases}$$

当 $N$ 为偶数时:

$$H(k) = \begin{cases} |H(k)| e^{-j\frac{2\pi}{N}\left(k+\frac{1}{2}\right)\left(\frac{N-1}{2}\right)} & k = 0, \dots, \left(\frac{N}{2}-1\right) \\ |H(N-1-k)| e^{j\frac{2\pi}{N}\left(N-k-\frac{1}{2}\right)\left(\frac{N-1}{2}\right)} & k = \frac{N}{2}, \dots, N-1 \end{cases}$$



频率响应:

$$H(e^{j\omega}) = e^{-j\left(\frac{N-1}{2}\right)\omega} \left\{ H_{\frac{N-1}{2}}(\omega) + \sum_{k=0}^M \frac{|H(k)|}{N} \left[ \frac{\sin \left\{ N \left[ \frac{\omega}{2} - \frac{\pi}{N} \left( k + \frac{1}{2} \right) \right] \right\}}{\sin \left[ \frac{\omega}{2} - \frac{\pi}{N} \left( k + \frac{1}{2} \right) \right]} + \frac{\sin \left\{ N \left[ \frac{\omega}{2} + \frac{\pi}{N} \left( k + \frac{1}{2} \right) \right] \right\}}{\sin \left[ \frac{\omega}{2} + \frac{\pi}{N} \left( k + \frac{1}{2} \right) \right]} \right] \right\}$$

当 $N$ 为奇数时:

$$H_{\frac{N-1}{2}}(\omega) = \frac{\left| H \left( \frac{N-1}{2} \right) \right|}{N} \cdot \frac{\cos \left( \frac{\omega N}{2} \right)}{\cos \left( \frac{\omega}{2} \right)} \quad M = \frac{N-3}{2}$$

当 $N$ 为偶数时:  $H_{\frac{N-1}{2}}(\omega) = 0$   $M = \frac{N}{2} - 1$

# 5、过渡带抽样的优化设计

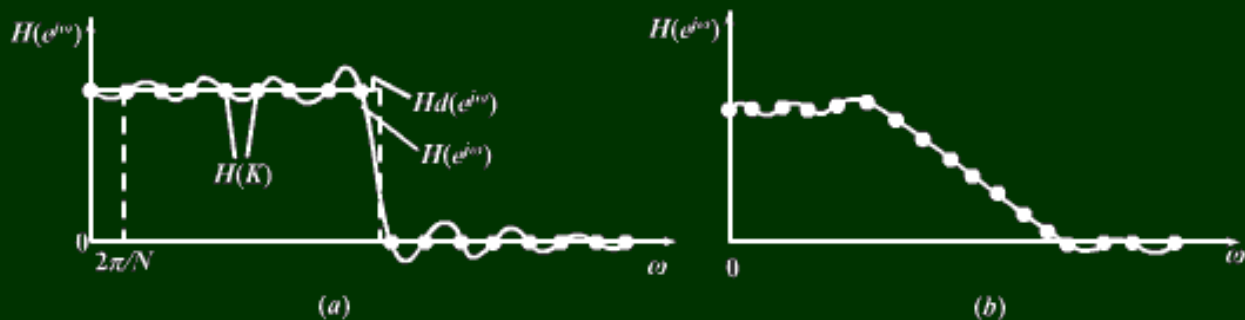


图7-16 频率抽样的响应

增加过渡带抽样点，可加大阻带衰减

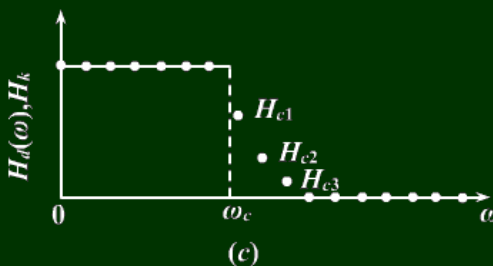
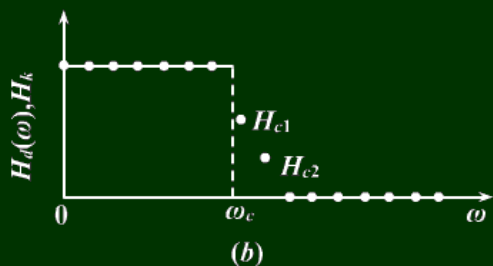
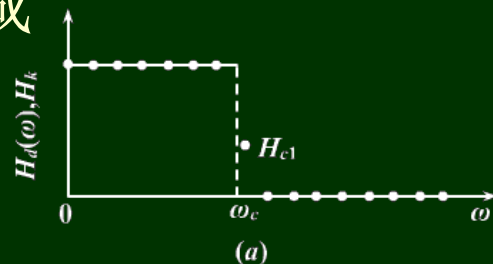
$$H(e^{j\omega}) = \sum_{k=0}^{N-1} H(k) \Phi\left(\omega - \frac{2\pi}{N}k\right)$$

不加过渡抽样点： $\delta_2 = -20dB$

加一点： $\delta_2 = -40 \sim -54dB$

加两点： $\delta_2 = -60 \sim -75dB$

加三点： $\delta_2 = -80 \sim -95dB$





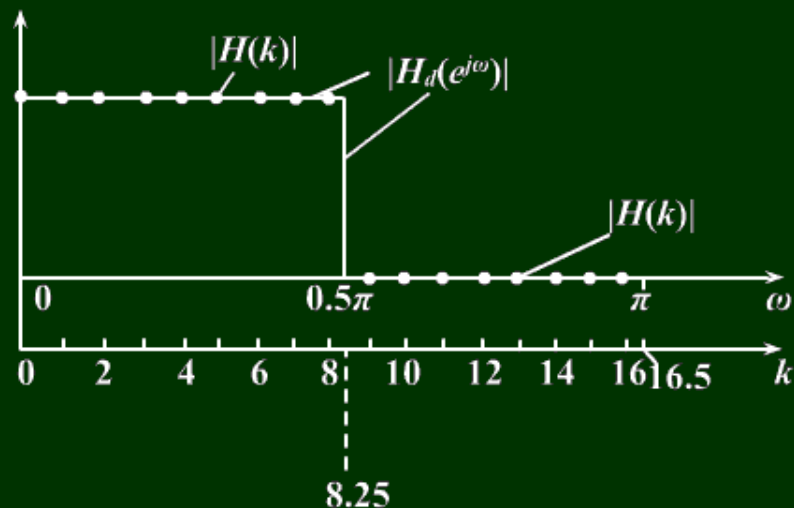
- ◆ 增加过渡带抽样点，可加大阻带衰减，但导致过渡带变宽
- ◆ 增加 $N$ ，使抽样点变密，减小过渡带宽度，但增加了计算量
- ◆ 优点：频域直接设计
- ◆ 缺点：抽样频率只能是  $2\pi/N$  或  $\pi/N$  的整数倍，截止频率  $\omega_c$  不能任意取值



例：利用频率抽样法设计一个频率特性为矩形的理想低通滤波器，截止频率为 $0.5\pi$ ，抽样点数为 $N=33$ ，要求滤波器具有线性相位。

解：理想低通频率特性

$$|H_d(e^{j\omega})| = \begin{cases} 1 & 0 \leq \omega \leq \omega_c \\ 0 & \text{其它 } \omega \end{cases}$$



按第一种频率抽样方式， $N=33$ ，得抽样点

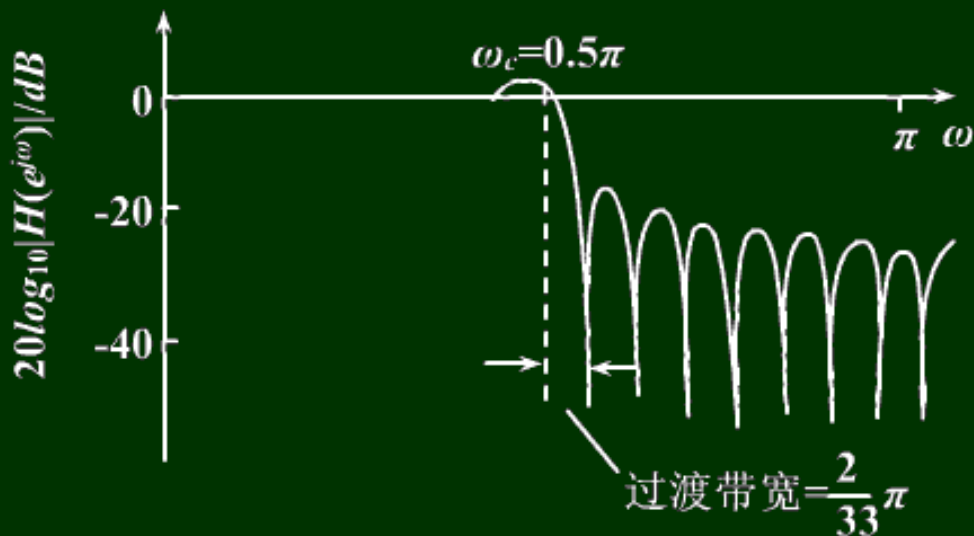
$$|H(k)| = \begin{cases} 1 & 0 \leq k \leq \text{Int} \left[ \frac{N\omega_c}{2\pi} \right] = \frac{N-1}{4} = 8 \\ 0 & \text{Int} \left[ \frac{N\omega_c}{2\pi} \right] + 1 = 9 \leq k \leq \frac{N-1}{2} = 16 \end{cases}$$

得线性相位FIR滤波器的频率响应:

$$H(e^{j\omega}) = e^{-j16\omega} \left\{ \frac{\sin\left(\frac{33\omega}{2}\right)}{33\sin\left(\frac{\omega}{2}\right)} + \sum_{k=1}^8 \left[ \frac{\sin\left[33\left(\frac{\omega}{2} - \frac{k\pi}{33}\right)\right]}{33\sin\left(\frac{\omega}{2} - \frac{k\pi}{33}\right)} + \frac{\sin\left[33\left(\frac{\omega}{2} + \frac{k\pi}{33}\right)\right]}{33\sin\left(\frac{\omega}{2} + \frac{k\pi}{33}\right)} \right] \right\}$$

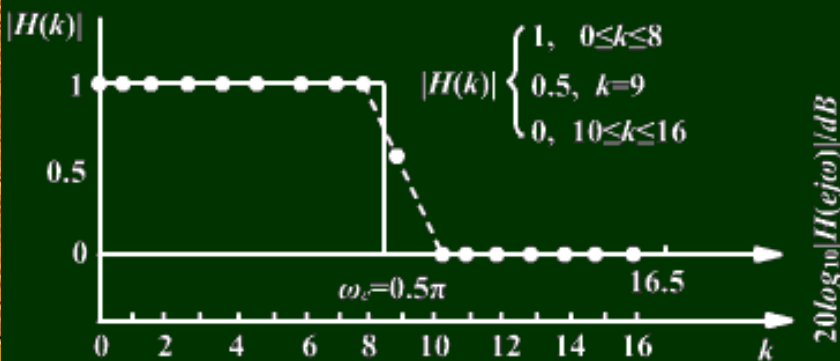
过渡带宽:  $2\pi/33$

阻带衰减:  $-20dB$

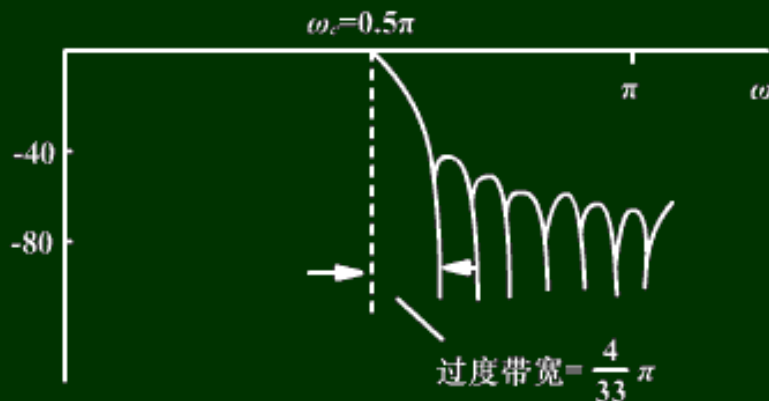


◆ 增加一点过渡带抽样点

令  $H(9)=0.5$



(a)



(b)

图7-20 增加过渡带非零抽样值及其影响  
(a)要求的频率响应 $|H_d(e^{j\omega})|$ 及其抽样 $|K(k)|$ ; (b)设计结果

过渡带宽:  $4\pi / 33$

阻带衰减:  $-40dB$

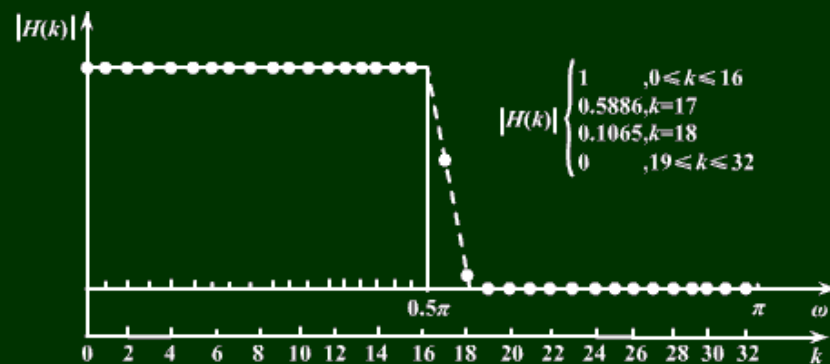
- ◆ 增加两点过渡带抽样点  
且增加抽样点数为 $N=65$

$$\text{令 } H(17)=0.5886$$

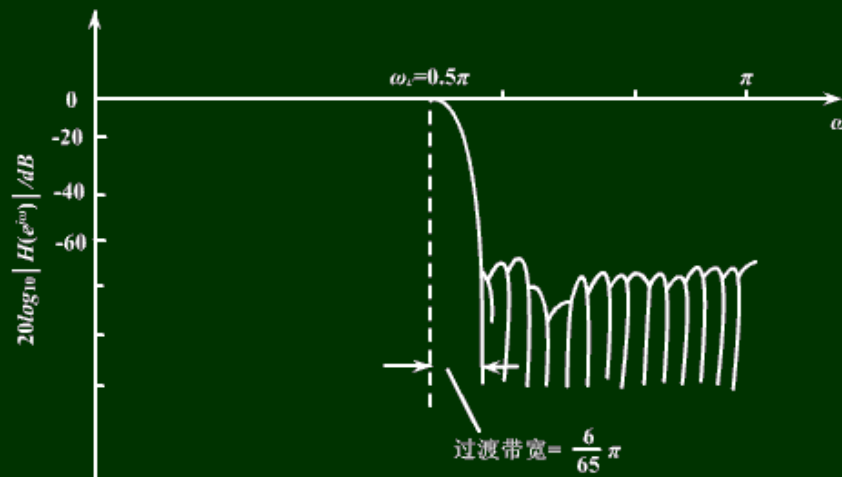
$$H(18)=0.1065$$

过渡带宽:  $6\pi / 65$

阻带衰减:  $-60dB$



(a)



(b)

图7-21 增加抽样点数 $N$ 及过渡带频率抽样值的情况  
(a)要求的频率响应 $|Hd(e^{j\omega})|$ 及其抽样 $|H(k)|$ ; (b)设计结果

## 五、IIR和FIR数字滤波器的比较

### IIR滤波器

- ◆  $h(n)$ 无限长
- ◆ 极点位于 $z$ 平面任意位置
- ◆ 滤波器阶次低
- ◆ 非线性相位
- ◆ 递归结构
- ◆ 不能用 $FFT$ 计算
- ◆ 可用模拟滤波器设计
- ◆ 用于设计规格化的选频滤波器

### FIR滤波器

- ◆  $h(n)$ 有限长
- ◆ 极点固定在原点
- ◆ 滤波器阶次高得多
- ◆ 可严格的线性相位
- ◆ 一般采用非递归结构
- ◆ 可用 $FFT$ 计算
- ◆ 设计借助于计算机
- ◆ 可设计各种幅频特性和相频特性的滤波器

